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MAGNETIC AMPLIFIER CHARACTERISTICS NEUTRAL TYPE.

BY

ALAN S. FITZGERALD.

GENERAL.

The material presented herewith comprises performance data relating specifically to magnetic amplifiers of the simplest form, intended mainly for action of the "On" and "Off" variety. It does not cover amplifiers designed to give a linear response. Likewise, it does not deal with high sensitivity amplifiers of the polarized type.

Thus the class of device referred to is best considered as a form of sensitive relay. In comparing the magnetic amplifier with existing types of sensitive relay the following reference magnitudes are of interest.

Available models of sensitive relay of the moving armature type, which have direct acting contacts, have a minimum response power level of about 10 to 20 milliwatts or 10 to 13 VU. Such relays list at from $5 to $10.

For sensitivity greater than these figures, existing types of sensitive relay are mostly of the moving coil type adapted from measuring instrument structure.

One such instrument, somewhat widely used, consists of a contact-making milliammeter of 4 ma range usually set to operate with a difference of 1 ma. This instrument has a resistance of 100 ohms. This would correspond to a power level of 100 microwatts, or −10 VU. Still more sensitive relays of this type are now obtainable which will operate with less than one microwatt, or −30 VU.

The contacts of these sensitive relays are not, in general, capable of controlling work devices of any substantial magnitude and they are usually used in conjunction with auxiliary relays. In the case of sen-

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sitive relays of the contact-making instrument type, some form of contact-sealing arrangement is in almost every case necessary, and accordingly sensitive relays of this type are not self-resetting. Such arrangements list at from $50 to $100.

With one exception, all of the magnetic amplifiers herein described actuate directly a simple and inexpensive moving armature relay of a substantial and rugged type, designed to be actuated by a power input of between one-tenth and one quarter of a watt. The minimum power level of the input to the amplifier at which this type of relay is operated, for the several amplifiers, lies in the range of from 6 microwatts to 0.1 microwatt, or from −22.2 VU to −40 VU.

The principal object in securing the accompanying data was to obtain more complete information as to the performance of a number of different types of amplifier in order to get a better idea of the applications for which they might be suitable.

DESCRIPTION OF AMPLIFIERS.

The group of amplifiers tested comprises one single-stage amplifier, four two-stage amplifiers, and three three-stage amplifiers. Three of these amplifiers operate on a frequency of 60 cycles, four of them at 720 cycles, and one at a combination of both of these frequencies. All of the circuit arrangements are based upon the compensating winding principle described in a previous paper.[1]

The cores of all the stages in any one amplifier consist of the same punching and the same stacking.

The amplifiers are identified by arbitrary numerals for reference purposes. It is convenient first to consider amplifier No. 8 because this was the first one constructed. Fig. 1 is a diagram of the connections of this amplifier. This diagram is the same as Fig. 12 of the paper referred to above and will not, therefore, be described in further detail here.

Amplifier No. 1 was built to find out what performance could be obtained with a three-stage 60 cycle amplifier similar to No. 8 but of reduced weight, size, and cost. Thus, No. 8 involves 2.5 lbs. of high permeability alloy having a base price of approximately $3 per lb. No. 1 has 1.52 lbs. of similar material. No. 8 is based upon the EI-3A lamination; EIM-7 is used in No. 1.[2] The connections of No. 1 are the same as those of amplifier No. 8 as shown in Fig. 1.

Amplifier No. 3 was built to obtain data regarding the improvement in performance, and reduction in weight and cost, which result from the use of a frequency higher than 60 cycles. Amplifier No. 3 is a two-stage 720 cycle amplifier using the EIM-7 core.

Amplifier No. 5 is a duplicate of No. 3 in all respects except for the input winding. No. 3 has a high resistance input winding suitable for receiving a few micro-amps current from an electron-emission type of phototube. No. 5 has a lower input resistance intended for use with a blocking-layer type photo-cell.

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[1] JOURNAL OF THE FRANKLIN INSTITUTE, **244**, 249.

[2] Allegheny Ludlum Steel Corporation lamination types.

The connections of amplifiers No. 3 and No. 5 are similar to the arrangement shown in Fig. 1 except that there are only two stages instead of three.

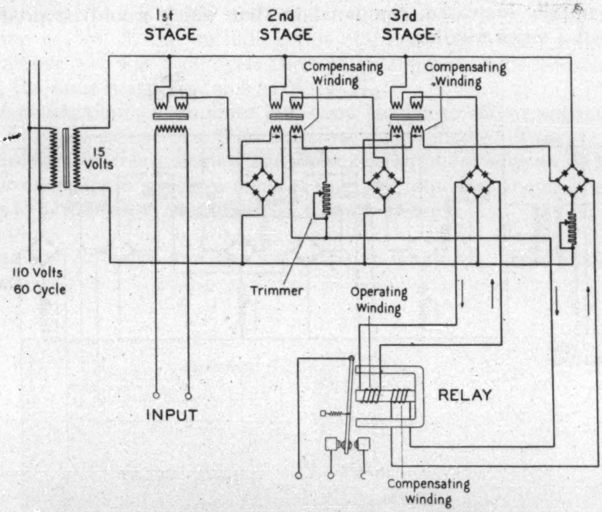
1st STAGE
2nd STAGE
3rd STAGE
Compensating Winding
Compensating Winding
15 Volts
110 Volts 60 Cycle
Trimmer
Operating Winding
RELAY
INPUT
Compensating Winding

FIG. 1. Three-Stage Neutral Type Magnetic Amplifier.

Amplifiers Nos. 1, 3, 5 and 8 are of the neutral type. That is to say, their response is independent of the polarity of the direct current input.

By the use of an additional direct current saturating winding connected in series with the rectified output current, as shown in Fig. 2,

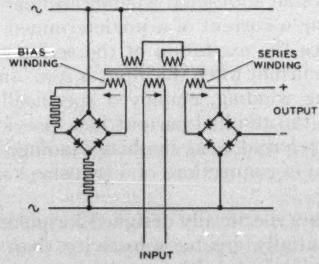
~
BIAS WINDING
SERIES WINDING
+
OUTPUT
−
~
INPUT

FIG. 2. Use of feed-back winding in magnetic amplifier.

two additional characteristics are produced. In the first place, the response is now different according to the polarity of the input. In the second place the gain is increased. If the input excitation opposes that of the series or feed-back winding, the output current is decreased rather than increased by the input. If the polarity of the input is such that the input and series windings are additive, then the output is substantially increased, compared to that which would be produced without a series winding.

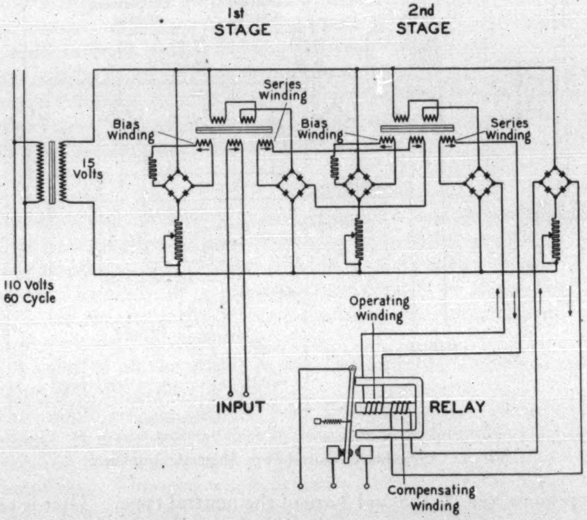
1st STAGE
2nd STAGE
Series Winding
Bias Winding
Bias Winding
Series Winding
15 Volts
110 Volts 60 Cycle
Operating Winding
INPUT
RELAY
Compensating Winding

Fig. 3. Two-stage magnetic amplifier with feed-back.

It is to be noted that even in a first or input stage, the use of a feed-back winding requires an additional winding, similar to a compensating winding, and carrying a current of a predetermined appropriate value. This winding opposes the excitation of the series winding due to the normal magnetizing current when the input is zero, and, as distinguished from a compensating winding, employed specifically for the purpose of compensating for the residual current in a previous stage, is, in a first or initial stage, referred to as the bias winding.

A typical diagram of connections of a two-stage amplifier with feed-back is shown in Fig. 3.

Magnetic amplifiers specifically designed for polarized response, and which are of substantially greater sensitivity than the arrangements referred to now, will be dealt with in a separate paper.

In the case of the amplifiers described in the present paper, feed-back is used for increasing the gain and not for the purpose of obtaining polarized response. Thus amplifiers which have feed-back windings in the first stage are only intended for use with the input, as, for example, a photo-cell, connected for the correct polarity; that is to say, the DC input current should aid the series winding.

Amplifier No. 2 is a 720 cycle unit having magnetic cores exactly the same as No. 3, but including this additional feed-back feature.

Amplifier No. 4 is a 60 cycle three-stage amplifier with feed-back, having the same magnetic cores as No. 1.

The connections of amplifier No. 4 are similar to the arrangement shown in Fig. 3 except that there are three stages instead of two.

Amplifier No. 6 is a single-stage, 720 cycle amplifier with an EIM-7 core for operating a sensitive relay of the type requiring about 10 milliwatts to actuate it. Amplifier No. 6 has feed-back.

Amplifier No. 7 is a two-stage amplifier in which the first stage operates at 720 cycles and the last stage at 60 cycles. Feed-back is included.

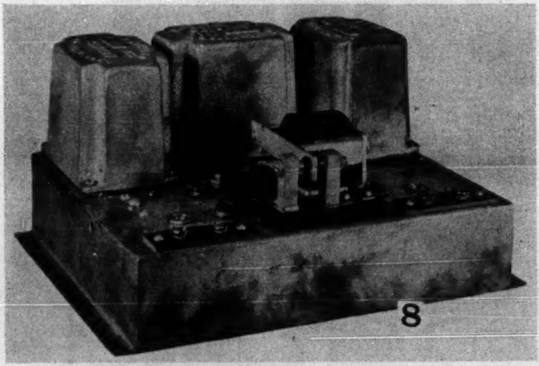

FIG. 4. Three-stage 60 cycle magnetic amplifier with EI-3A cores.

Fig. 4 is a photograph of amplifier No. 8. Fig. 5 is a picture of amplifier No. 4. Amplifier No. 1 is identical in appearance with amplifier No. 4. Fig. 6 is a view of amplifier No. 2. Amplifiers Nos. 2, 3, and 5 are of identical external appearance. Fig. 7 shows amplifier No. 6 and Fig. 8 is a view of amplifier No.7.

Fig. 9 is a picture of all five amplifiers together to show their relative sizes.

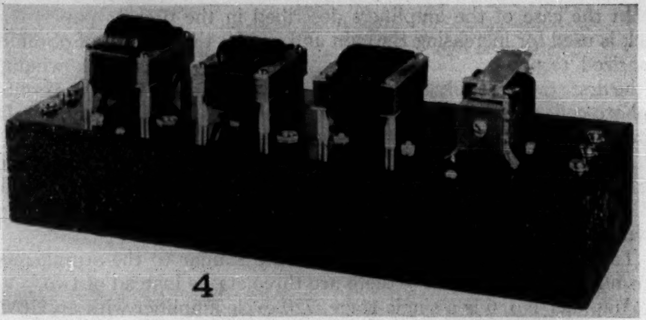

FIG. 5. Three-stage 60 cycle magnetic amplifier with EIM-7 cores.

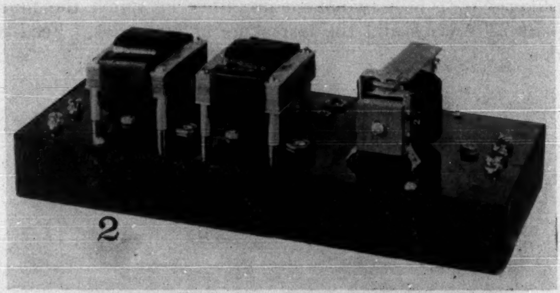

FIG. 6. Two-stage 720 cycle magnetic amplifier with EIM-7 cores.

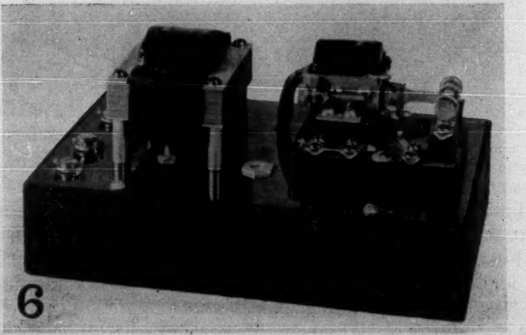

FIG. 7. Single-stage 720 cycle magnetic amplifier with EIM-7 core.

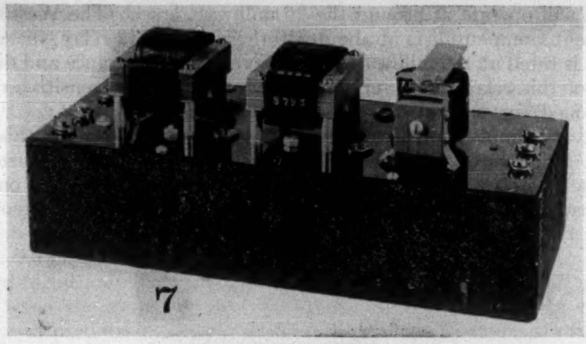

FIG. 8. Two-stage magnetic amplifier using both 720 and 60 cycles.

A tabulation, giving in compact form and in round numbers various items of data regarding the different amplifiers and their performances, is appended.

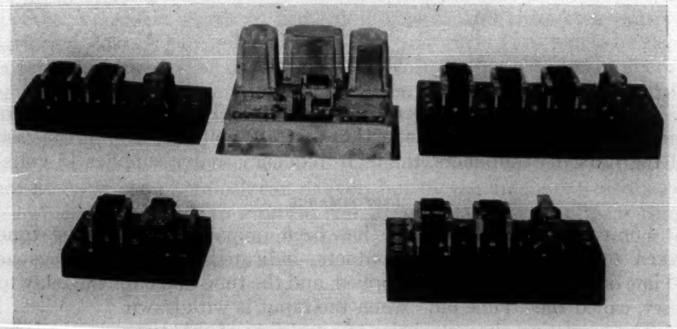

FIG. 9. Group of magnetic amplifiers.

RELAY.

Each amplifier includes a relay. The following data may be of interest in connection with the type of relay incorporated in all of the amplifiers with the exception of No. 6.

At the time this work was undertaken no relay of the characteristics desired appeared to be listed by any manufacturer. There are any number of relays which operate with about one watt, and quite a few sensitive relays having light moving parts and low contact pressures which will operate at around the 20 milliwatt level. The Weston Instrument Company lists at about $10 their model 712 relay, the coil of which is rated at 180 milliwatts. However, the appearance and dimensions of this relay differ very little from those of the sensitive relays mentioned above.

A very simple type of relay was therefore constructed for this purpose. Unmounted, it has a weight of 12 ounces and when adjusted to operate at 150 milliwatts it develops a contact pressure of 8 ounces. The back contact pressure de-energized is 1.75 ounces. The contact gap is 1/16 of an inch. This relay is illustrated in Fig. 10.

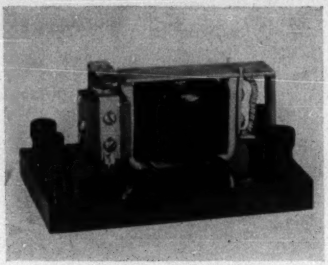

FIG. 10. Relay.

The amplifier circuits operate at 15 volts. The 60 cycle amplifiers include a transformer for connection to a 110 volt source. In the case of the 720 cycle amplifiers, the high frequency source supplies 15 volts.

PERFORMANCE.

For each amplifier a curve has been prepared showing the time taken to operate the relay contacts, indicated on the drawings as "Time on," when the input is applied, and the time taken for the relay to reset, called the "Time off," when the input is withdrawn.

The operating characteristics of amplifiers of this type, particularly in respect to time of response, depend to a considerable extent upon the initial adjustment of the compensating current settings. Increasing the compensating current decreases the sensitivity, increases the closing time or "Time on" and decreases the opening time or "Time off."

Conversely, reducing the compensating current tends to increase the sensitivity and gives a somewhat higher speed of response on applying the input. However, when so adjusted the amplifier may be sluggish in its response when the input is withdrawn; that is to say, the opening time or "Time off" may be increased.

Thus in each of the curves showing operating time of individual amplifiers, an average adjustment is shown in full lines. Other settings giving faster "close" or faster "open" actions are shown in broken lines.

In addition a second curve sheet, for each unit, is furnished, showing the relation between the power applied to the input of the amplifier and the output of the amplifier; that is to say, the power developed in the operating winding of the relay. This output power is computed in the manner more fully explained in a previous paper.[3] Briefly, it consists of the amount of power which is developed in a resistance equal to that of the relay operating winding by a current equal to the difference between the current which flows in the winding when there is no input applied to the amplifier, and the current, of increased magnitude, which flows when the amplifier receives input. In other words, the power which will produce in a single relay winding of the same resistance and turns as the operating winding, excitation ampere-turns equal to that which the relay receives, when amplifier input is applied, due to the resultant effect of the current in the operating winding and the current in the compensating winding.

The ratio between the input and output, or gain, is also plotted on this sheet.

Data relating to all the seven types of amplifiers are also plotted together on single sheets to bring out more clearly their comparative performances. The combined operating time curves are plotted twice, once against the power input, and once with the input given in microamps. The latter information is of more value in connection with photo-electric cells of the electron-discharge type in which a high resistance input winding is used. In this instance, the maximum power which can be developed in the input winding of the amplifier is limited not by what the photo-cell can deliver, but by the maximum resistance for which it is possible to wind the coil. Thus magnetic amplifiers using large-size cores having greater winding space are at an advantage in respect to photo-cells of this type.

OPERATING TIME.

In considering possible applications for magnetic amplifiers it is evident that the most important single characteristic is the time of operation. Except in a somewhat limited field of application in which instantaneous action is not required, the response time of the magnetic amplifier would seem to be the principal limitation.

It is necessary, therefore, to give special consideration to what is meant by the operating time of a magnetic amplifier. In most practical applications which are encountered, the resultant of two elapsed periods has to be considered: first, the time which it takes for the amplifier to respond to the presence of current in the input winding; and second, the time which it takes to establish the said current in the highly inductive input circuit.

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[3] JOURNAL OF THE FRANKLIN INSTITUTE, **244**, 323.

Unfortunately, although amplifiers of fair speed of response can be furnished, the latter time element is very considerable.

Before studying the curves of the various amplifiers it is therefore important to consider the above relations. These are brought out clearly in Fig. 11.

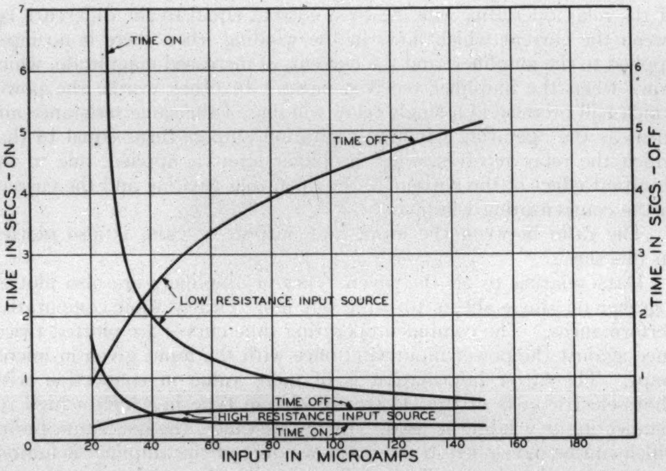

FIG. 11. Operating time of magnetic amplifiers. Effect of input circuit characteristics.

On this sheet the operating time of amplifier No. 5 had been taken two different ways. In one case, a high resistance is connected in series with the input winding. A voltage is applied to this high resistance circuit, including the input winding, and the elapsed time between the application of this voltage and the operation of the relay is measured. Under these circumstances the application of exciting ampere-terms to the core of the input stage is substantially instantaneous. With this "applied excitation" method of testing, the elapsed time which is measured represents the response time of the amplifier per se.

The operating times are again shown in Fig. 11 without any high resistance in the input circuit. The time which is now measured will be that which elapses between the application of the input voltage to the winding of the amplifier and the relay action, which, now, comprises the resultant of the response time of the amplifier and the time taken to establish the input excitation.

It will be noted that the time delay in the input circuit is substantially greater than the response time of the amplifier.

These circumstances are clearly of importance in considering the suitability of magnetic amplifiers for various applications. In some cases the first, and in other instances the second, of the operating conditions shown in Fig. 11 will exist.

PHOTO-ELECTRIC APPLICATIONS.

These characteristics are of special interest in connection with the use of magnetic amplifiers with photo-electric cells, as will be seen on referring to Fig. 12 and Fig. 13 which refer to amplifiers No. 3 and No. 5.

Amplifiers No. 3 and No. 5 are both of the two-stage 720 cycle type. The amplifiers are identical in every respect except that they have input windings of different resistance. No. 3 has a high resistance winding of 25,000 ohms, intended for use with a photo-electric cell of the electron-discharge type. Amplifier No. 5 has an input resistance of 4,000 ohms which is suitable for use with a photo-electric cell of the blocking-layer type.

A photo-electric cell of the emission type has a high resistance in the order of megohms and is used with a source of relatively high EMF. Thus the operating conditions which apply are those of the lower set of curves in Fig. 11.

The blocking-layer type of photo-electric cell, on the other hand, relatively, is a low EMF low resistance device and the conditions which obtain are those of the upper set of curves in Fig. 11.

The different effects of these two operating conditions are shown very clearly in Fig. 12 and Fig. 13 which show the performance of amplifier No. 3 with a gas type caesium cell PJ23, at 45 volts DC, and of amplifier No. 5 with a Photronic cell model 594, type 2, No. 699–60.

Both cells are subject to the same light condition, being illuminated at a distance of 10 inches by means of a 50 watt lamp the voltage of which is adjusted to give different values of illumination.

Fig. 12 shows the "On" and "Off" times of the two combinations with equivalent light conditions, and Fig. 13 compares the power output in a similar manner. It is of special interest to note that although the Photronic cell gives greater power in the relay winding under static conditions, the emission-type of cell gives a substantially higher rate of response, due to the difference in times taken to develop excitation in the input circuit.

Since the time delay in the input circuit, as distinguished from the time of response of the amplifier, is constant for a given input winding and core, irrespective of other features of the design of the amplifier (except when the first stage has a feedback winding), the comparative performances of the several different types of amplifiers are brought out more clearly by plotting curves on the basis of amplifier response time alone; that is, the high resistance input circuit condition.

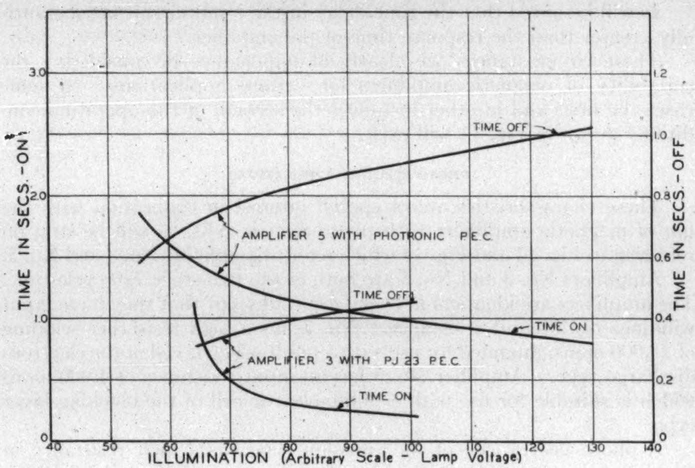

FIG. 12. Operating time of magnetic amplifiers.

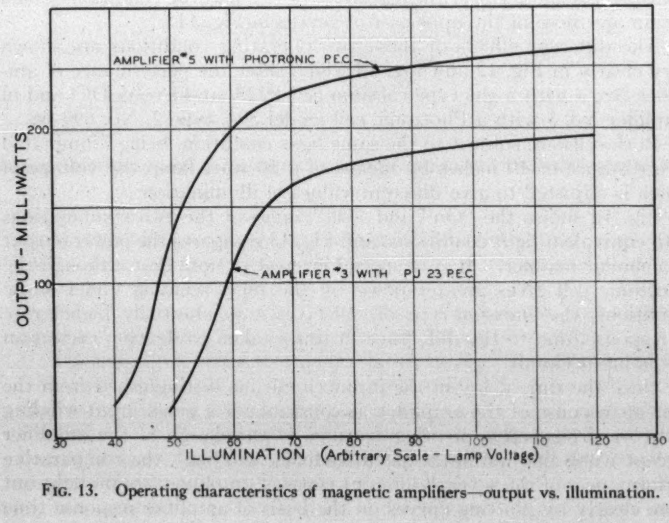

FIG. 13. Operating characteristics of magnetic amplifiers—output vs. illumination.

Accordingly, all of the amplifiers, with the aforementioned exception of No. 5, were wound with a high resistance input winding and were tested with a photo-cell of the electron-discharge type.

The operating times were obtained with an electric chronometer, the elapsed time being indicated from the instant of switching on the lamp. Thus the time of heating of the lamp is included in the total time.

All of the curves, therefore, refer to this type of action and this must be kept in mind in considering the suitability of these magnetic amplifiers for various applications.

The performance shown is based upon operating conditions such as are encountered in photo-electric applications, that is to say, when the unit cannot easily be subjected to input power exceeding a few hundred times the pick-up value.

In cases where the operating conditions are more severe in this respect, the amplifier should be adjusted so as to strengthen the resetting action and the sensitivity then obtained will be somewhat lower than the value shown by the curves.

For each of the seven types of amplifier (No. 5 being excluded since this is equivalent to No. 3 except for the input winding) the time curves and gain and output curves are shown together on the same page in Figs. 14 to 27 inclusive.

The combined operating times of the group of amplifiers are plotted together, against input current, in Fig. 28 "Time on" and in Fig. 29, "Time off."

The combined operating times plotted against input power in VU are similarly shown in Figs. 30 and 31.

Fig. 32 shows the output curves plotted together on a single sheet.

The relays in the various amplifiers were not all adjusted alike. For example, in amplifiers No. 2 and No. 8 the relay setting was around 250 milliwatts and was correspondingly lower in the case of the other amplifiers having a lower output.

A broken line is marked on Fig. 32 indicating the 150 milliwatt level (21.8 VU).

Incidentally, since we have been comparing magnetic amplifiers with sensitive relays, it is of interest to note that it is not to be assumed that the action of sensitive relays is in all cases instantaneous. Where the input power level is measured in milliwatts quick response is usually obtained. But the very sensitive moving coil relays, when the input is a small fraction of a microwatt, take several seconds to operate, in some instances longer than a magnetic amplifier.

RESULTS.

The following comments may be made in respect of the several different types of amplifiers tested.

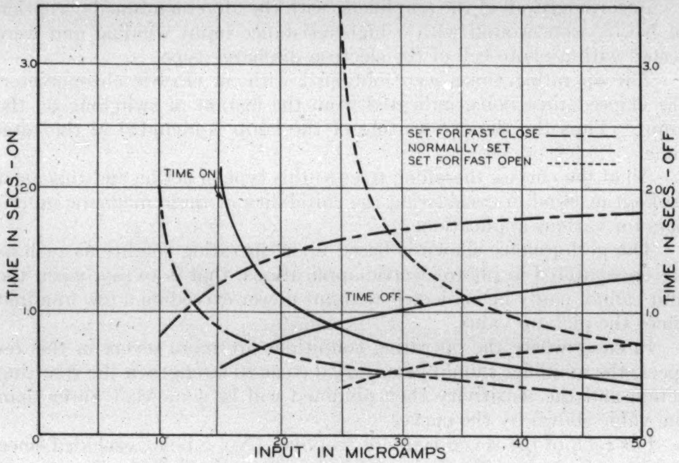

FIG. 14. Operating time of magnetic amplifier No. 1.

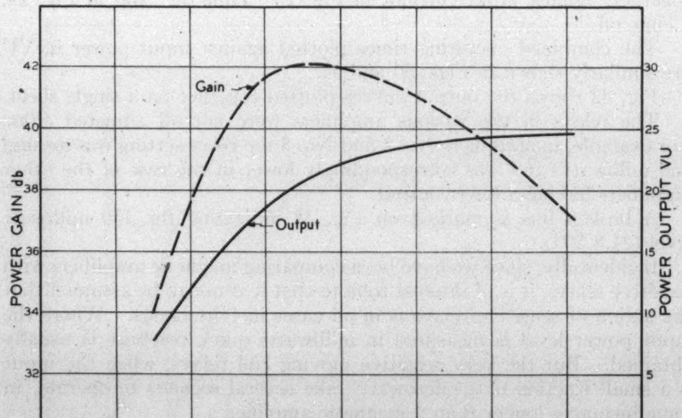

FIG. 15. Gain and output vs. input—magnetic amplifier No. 1.

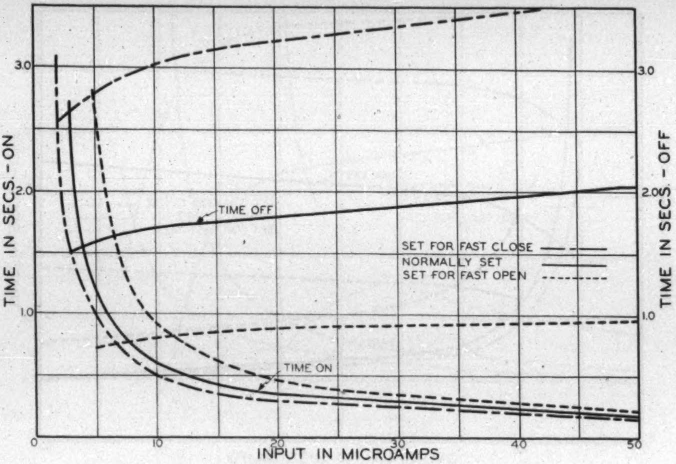

FIG. 16. Operating time of magnetic amplifier No. 2.

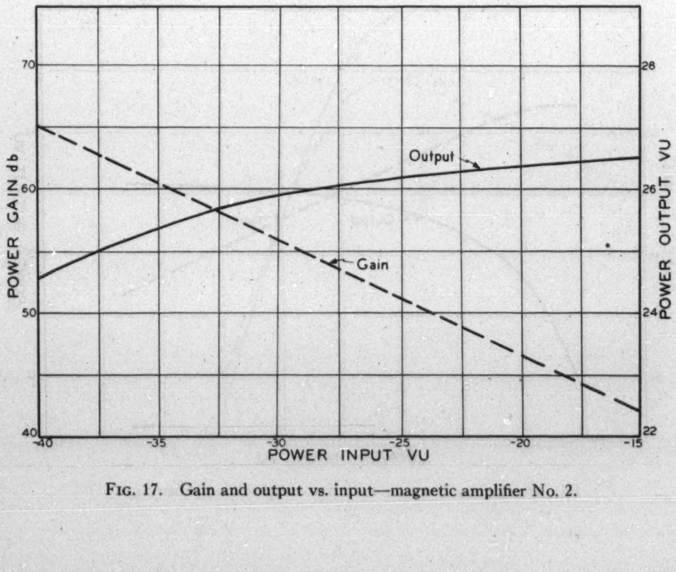

FIG. 17. Gain and output vs. input—magnetic amplifier No. 2.

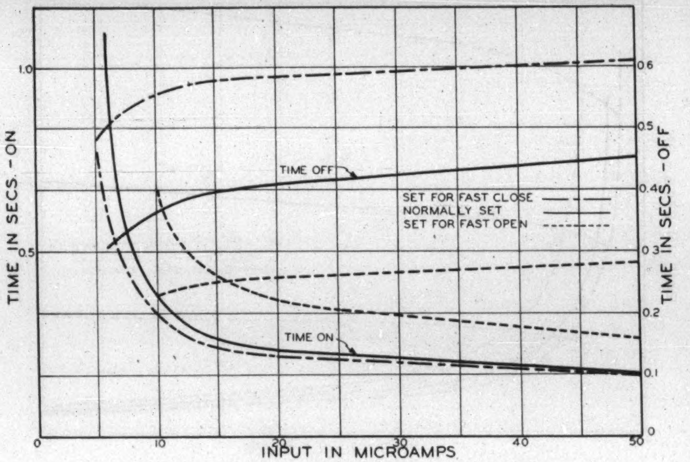

FIG. 18. Operating time of magnetic amplifier No. 3.

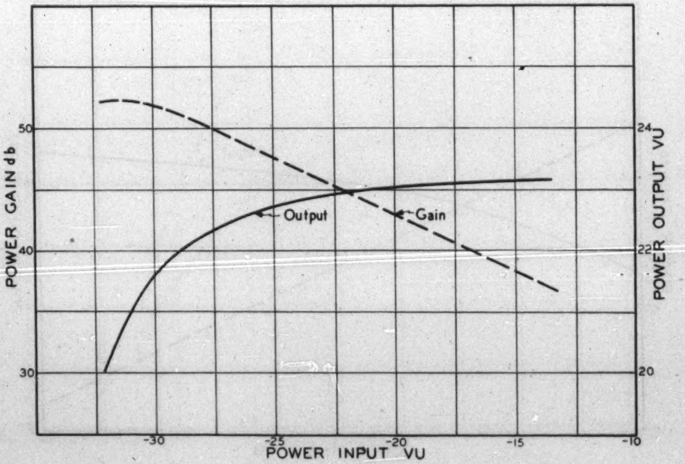

FIG. 19. Gain and output vs. input—magnetic amplifier No. 3.

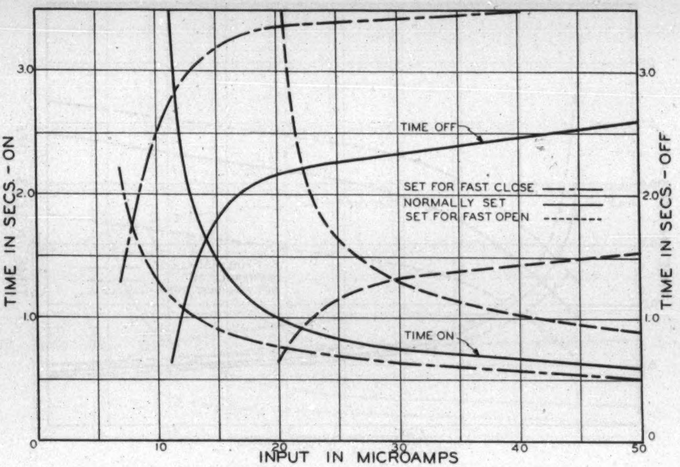

FIG. 20. Operating time of magnetic amplifier No. 4.

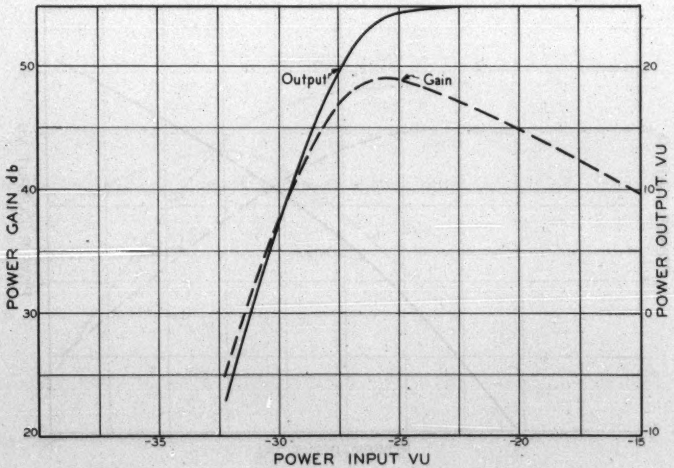

FIG. 21. Gain and output vs. input—magnetic amplifier No. 4.

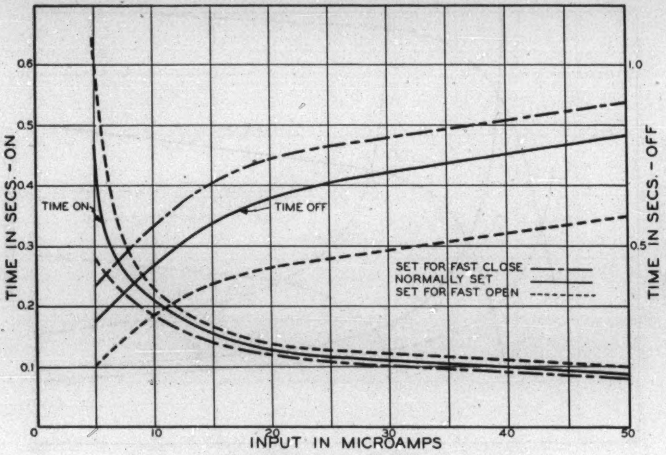

Fig. 22. Operating time of magnetic amplifier No. 6.

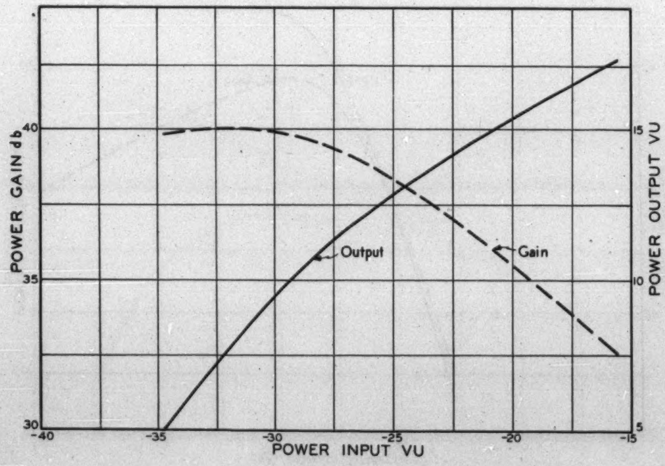

Fig. 23. Gain and output vs. input—magnetic amplifier No. 6.

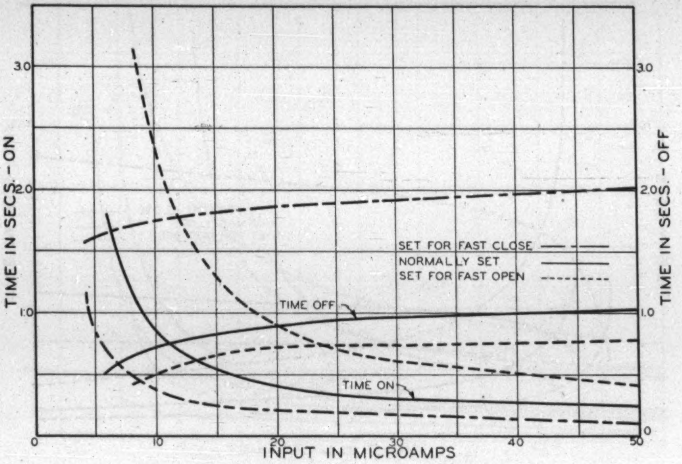

FIG. 24. Operating time of magnetic amplifier No. 7.

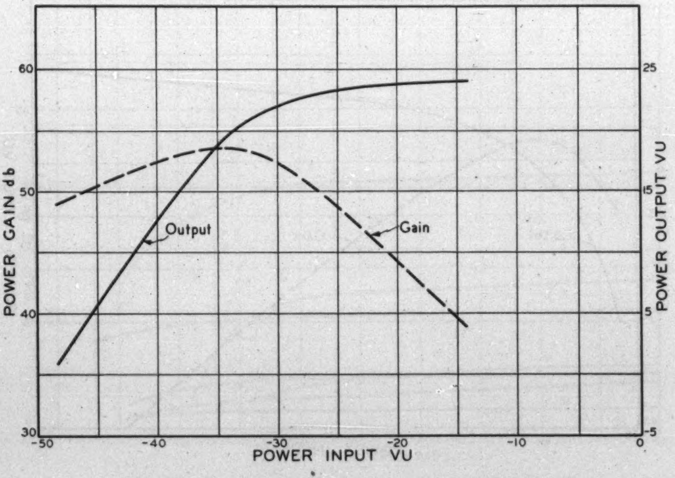

FIG. 25. Gain and output vs. input—magnetic amplifier No. 7.

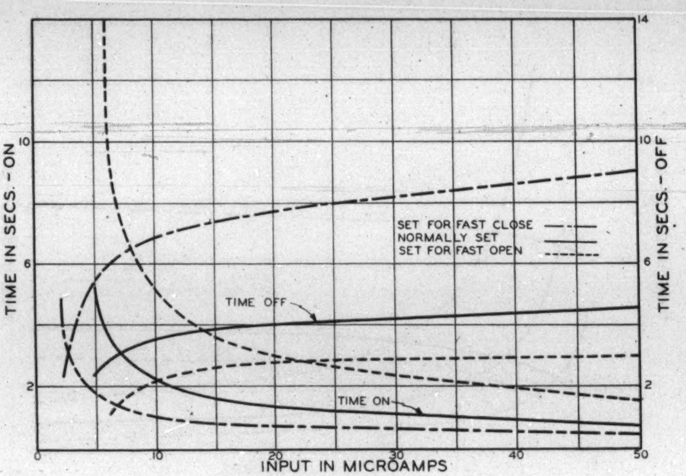

FIG. 26. Operating time of magnetic amplifier No. 8.

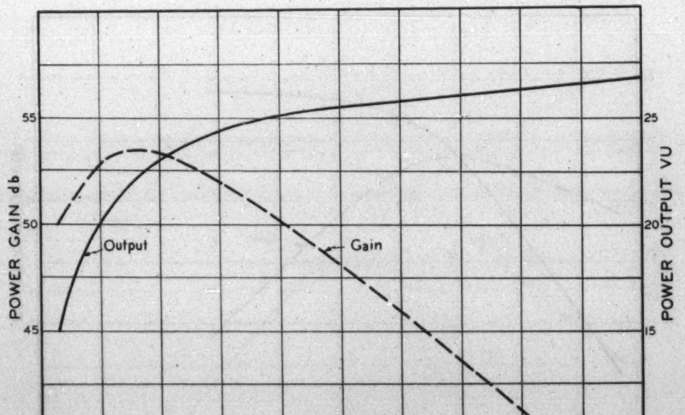

FIG. 27. Gain and output vs. input—magnetic amplifier No. 8.

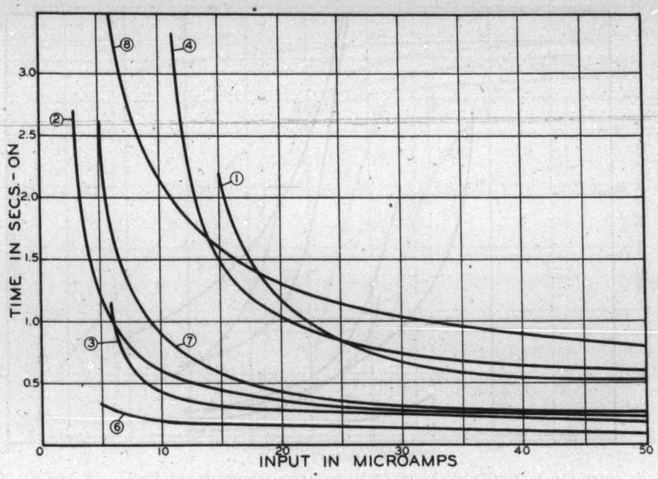

FIG. 28. Operating time of magnetic amplifiers.

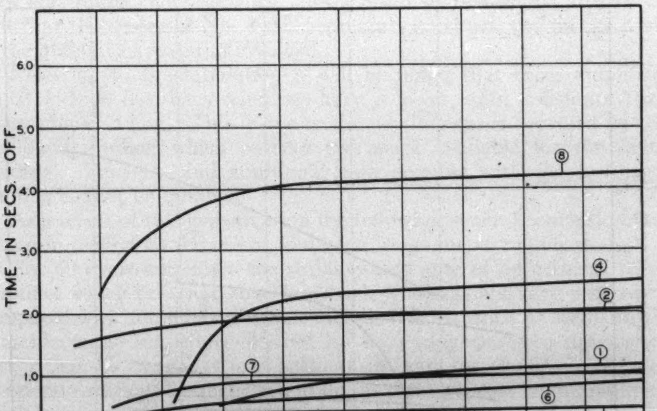

FIG. 29. Operating time of magnetic amplifiers.

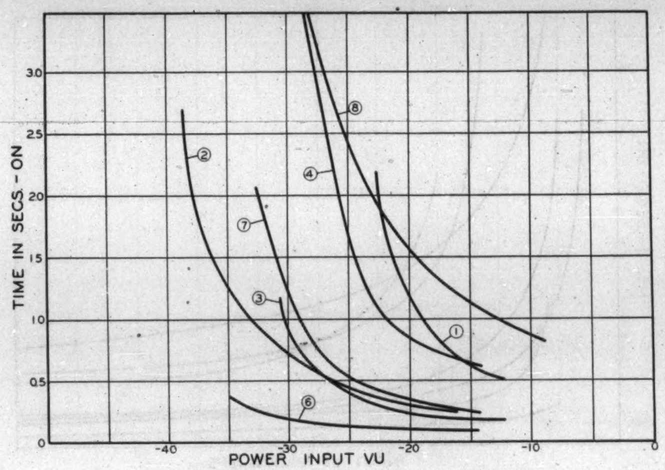

FIG. 30. Operating time of magnetic amplifiers.

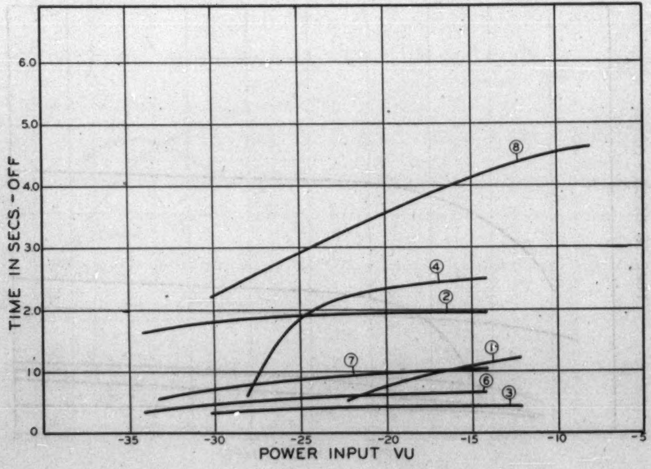

FIG. 31. Operating time of magnetic amplifiers.

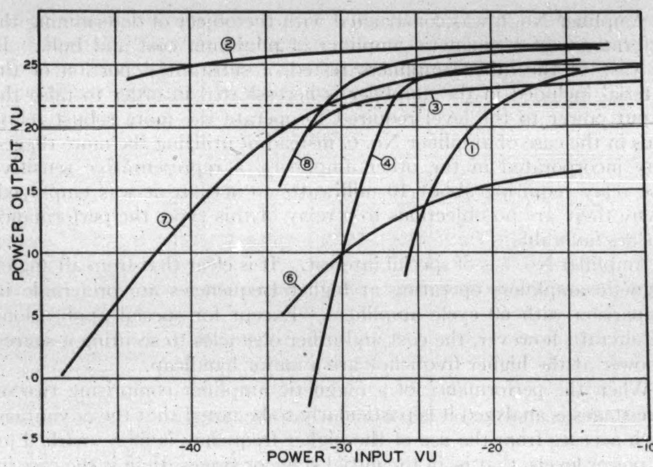

FIG. 32. Operating characteristics of magnetic amplifiers—output vs. input.

No. 8 will be mentioned first in that the unit was designed and built in 1936. As indicated above, it has been found to have good performance and stable characteristics. As a result of this study, it may be said that the design of No. 8 still represents good practice on the basis of the amount of material involved.

Referring to the tabulation it will be noted that those amplifiers which include feed-back windings have a lower input resistance than those which do not. This is due to the winding space occupied by the feed-back winding which reduces the space available for the input winding. Thus the full additional gain possible with the feed-back winding cannot be obtained.

As a result of this investigation the following general conclusions are drawn in respect to the use of feed-back in magnetic amplifiers.

For 60 cycle amplifiers the stability and ease of adjustment of an amplifier which does not have feed-back windings are very marked as compared with amplifiers which do include this feature. It seems better to achieve the sensitivity desired by increasing the core dimensions rather than by the use of feed-back, at any rate for amplifiers intended to operate without frequent attention, as distinguished from amplifiers for laboratory use. It will be noted, for example, that the microampere sensitivity of No. 8 is superior to that of No. 4. No. 8 is a thoroughly stable unit easy to adjust. No. 4, on the other hand, is highly critical, not very stable, and more susceptible to line voltage fluctuations.

Amplifier No. 6 was constructed with the object of determining the performance of a magnetic amplifier of minimum cost and bulk. In the case of the other amplifiers tested, a substantial portion of the material included in the amplifier is necessitated in order to raise the output power to the level required to operate the more robust relay. Thus in the case of amplifier No. 6, instead of utilizing the more rugged relay incorporated in the other amplifiers, a representative sensitive type relay requiring about 10 milliwatts to actuate it was employed. Where there are no objections to a relay of this type, the performance appears favorable.

Amplifier No. 7 is of special interest. It is clear that from all angles magnetic amplifiers operating at higher frequencies are preferable in comparison with 60 cycle amplifiers. Except for special applications like aircraft, however, the cost and other obstacles to securing a source of power at the higher frequency are a major handicap.

When the performance of a magnetic amplifier comprising two or more stages is analyzed it is particularly to be noted that the advantage which accrues from the use of the higher frequency is more marked at low power levels, that is, in the initial stage or stages, than is the case in subsequent stages when the power has been stepped up to substantial levels.

One of the major advantages resulting from the use of higher frequencies is improvement in the speed of response. The major portion of the response time of the amplifier occurs in the initial stage or stages. That is to say, the time varies inversely with the stage power level. In other words, the difference between 60 cycle and 720 cycle speed of operation is much greater in the first stage than in the last stage.

Considering now the amount of power needed to operate the amplifier, that is to say, the VA drain on the supply source, it is seen that very little of this energy is associated with the active stage output current in the initial stage. A major portion of the supply drain consists of the output of the amplifier, that is, the power supplied to the relay, and a further substantial quota is expended in the several bias or compensating windings both in the relay and the latter stage or stages.

The above considerations logically lead to the arrangement adopted in the case of amplifier No. 7 in which the first stage only is operated at 720 cycles. The second stage and all of the auxiliary windings are supplied at 60 cycles.

When the performance of No. 7 is compared, for example, with that of No. 3 (720 cycles) and No. 1 (60 cycles), it is very clearly seen that the performance of No. 7 much more nearly approximates the former than the latter.

On the other hand, the 720 cycles power consumption of No. 3 is a 4 VA whereas that of No. 7 is 0.4 VA.

The results obtained with No. 7 suggest that, in view of the extremely small 720 cycle power requirements, some way of devising an individual 720 cycle source of this limited magnitude, to be included in the amplifier unit, may be worth investigating.

SUMMARY.

Amplifier No.	1	2	3	4	5	6	7	8
Frequency	60	720	720	60	720	720	60/720	60
No. of Stages	3	2	2	3	2	1	2	3
Punching	EIM7	EIM7	EIM7	EIM7	EIM7	EIM7	EIM7	EI3A
Feed-back	No	Yes	No	Yes	No	Yes	Yes	No
Resistance of Input Winding	25,000	14,000	25,000	14,000	4,000	14,000	14,000	43,000
Minimum Micro-amps Input to Operate Relay	15	3	6	11	19	5	6	5
Minimum Micro-watts Input to Operate Relay	6	0.13	0.9	1.7	0.9	0.3	0.5	1.1
Maximum Power Gain (Output/Input)	16,200	4,460,000	170,000	91,000	170,000	10,000	224,000	240,000
Operating Time in Seconds with Input of 10 Micro-watts	1.15	0.30	0.25	0.85	0.25	0.10	0.35	0.60
60 Cycle VA Consumption	14	—	—	23	—	—	23	12
720 Cycle VA Consumption	—	4	4	—	4	2	0.4	—
Weight in Pounds of High Permeability High Cost Core Material	1.5	1.5	1.5	1.5	1.5	0.75	0.75	2.5
Total Weight in Pounds of Complete Amplifier and Relay*	7	4	4	7	4	2	5	15

* Uncased and unmounted except No. 8 which has cased reactors chassis mounted.

Isotope Separation by Electromigration. (*Electrical Engineering*, Vol. 66, No. 5.)—A method of separating isotopes by counter-current electromigration has been developed by the National Bureau of Standards in connection with work on the natural isotopes of potassium and chloride. This method makes use of the difference in the ionic mobilities of the isotopes of an element, and has been developed to a point where it can be used as a practical means of separating not only isotopes, but ionic species in general.

The entire process takes place within an electrolytic cell containing anode, cathode, and suitable electrolyte. The basic principle of operation of the method rests in an imposed flow of electrolyte through the solution in a direction opposite to the migration of the ions to be separated. If the rate of flow of the electrolytic counter-current is made equal and opposite to the average rate of ion transport, then only the lighter isotopic ions will make headway against the current, while the ions of the heavier isotope are carried in the opposite direction.

In the separation of electrolytic ions, where ion transport can be looked upon as a small forward drift super imposed upon kinetic agitation, it is necessary to reduce the rate of remixing resulting from convection in the electrolyte to a point below the rate of ion separation. Also, to obtain appreciable concentrations of a given isotope, a multistage fractionation process must be employed. Both of these requirements are met by carrying on the electrolysis in a fine-grained packing of uniform porosity. The packing not only reduces remixing to a minimum, but acts as a fractionation column under total reflux.

The process first was used in the separation of the isotopes of potassium. For this purpose an electrolytic cell consisting essentially of vertical cathode and anode compartments (provided with droppers and funnels for addition of H_2SO_4 and KOH, respectively), is immersed in a mechanically agitated constant-temperature bath. The cell also is provided with a funnel for the addition of the K_2SO_4 electrolyte. Electrodes are of platinum gauze. A siphon connects the anode compartment to a constant-level spillover. Addition of further liquid to the solution thus causes a flow of electrolyte through the packing in the horizontal connecting tube toward the anode in a direction opposite to the migration of the positive ions.

The counter-current electromigration method has several inherent advantages over alternative methods of isotope separation that have been used in recent years. The simplicity of the apparatus, both in construction and operation, is perhaps its most important feature. Isotopic separation takes place in a single step without the need of a vacuum system, or other elaborate accessories. The system requires very little attention, and with the use of automatic controls for addition of restituent liquids it becomes entirely self-regulating. The use of a very fine uniform porous packing provides a much higher reflux efficiency per unit length than may be achieved by other fractionation methods of isotope separation. Hence, the assembly has the advantage of requiring a minimum of space for the results accomplished. The process has the added convenience of being well adapted for use with many elements which may be obtained easily in ionic solutions.

R. H. O.

ANALYTIC REPRESENTATION OF EXPERIMENTAL SPECTRORADIOMETRIC CURVES.

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1. INTRODUCTION.

It is a strange fact that theoretical photometry and colorimetry have been developed on a numerical rather than an analytic basis. All the photometric and colorimetric quantities are obtainable by weighting physical measurements by means of the standard data for the human eye. But these weighting functions have always been given in the form of tables instead of equations. Thus in integrating, the investigator has been forced to use clumsy numerical methods; and this numerical approach has become so firmly established that most colorimetrists do not even realize that there is an alternative.

The only place in the entire subject where analytic methods are used is in the treatment of blackbody radiation. The value of even this slight trace of analyticity is evidenced by the wide use of the Wien and Planck equations even in places where they do not apply. Our aim has been to show that analytic methods can be employed throughout photometry and colorimetry.[1] In previous papers, we showed that the standard tables of *weighting factors* can be approximated by Pearson functions.[1,2] The feasibility of employing polynomials to approximate the common types of *reflectance* and *transmittance* curves was also considered.[3] Turning now to spectroradiometric curves for light sources, we have found that Planckian radiation can be approximated very simply in the visible region by a three-term polynomial, and somewhat more exactly by a Wien function.[4] The present paper continues this work by a consideration of experimentally determined spectroradiometric curves for light sources.

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[1] Parry Moon and D. E. Spencer, *J. Opt. Soc. Am.*, **33**, 89 (1943); "Analytic Expressions in Photometry and Colorimetry," *J. Math. and Physics*, **25**, 111 (1946).

[2] Moon and Spencer, "Analytical Representation of Trichromatic Data," *J. Opt. Soc. Am.*, **35**, 399 (1945).

[3] Moon and Spencer, "Polynomial Representation of Reflectance Curves," *J. Opt. Soc. Am.*, **35**, 597 (1945).

[4] Moon and Spencer, "Approximations to Planckian Distributions," *J. App. Phys.*, **17**, 506 (1946).

The light sources of greatest importance in illuminating engineering may be classified as

(a) Sun and sky,
(b) Incandescent lamps,
(c) Electric discharge lamps producing visible radiation directly (mercury, neon, sodium lamps),
(d) Fluorescent lamps.

Spectral-distribution data are available for the various phases of daylight and for numerous lamps. These experimental curves, however, constitute a cumbersome method of specification and one that is not directly amenable to mathematical manipulation. It is found that curves for (a) and (b) can be approximated analytically, in the visible spectrum and within the limits of experimental error, by simple polynomials. The curves for fluorescent lamps (d) are represented by Pearson functions. For the line spectra of (c), specification is simple and only summation, not integration, is required. So (c) needs no further consideration.

It is now possible to represent spectroradiometric curves for light sources in a practical analytic form and to perform all necessary photometric and colorimetric integrations analytically. Details are given in the following sections. Since the trichromatic weighting functions apply only to the visible spectrum, the analytic representations are confined to this region and no attempt is made to cover the ultraviolet or infrared.

2. DAYLIGHT.

This section deals with

(1) Direct sunlight,
(2) Light from an overcast sky,
(3) Light from zenith sky on a clear day.

Other daylight conditions can be handled in a similar manner. Many data are available on direct sunlight. Most information is based on the very extensive researches of the Smithsonian Institution, though work has been done also by Fabry and Buisson, Wilsing, Pettit, and others. Figure 1 shows a recommended curve[5] for the direct solar irradiation of a surface that is normal to the sun's rays. The sun is at a zenith angle of 60°, the observer is at sea level, and the atmosphere contains an ordinary amount of dust and water vapor.[5]

The heavy curve of Fig. 1 was approximated by a three-term polynomial:

$$J(\lambda) = K_0 - K_1\lambda - K_2\lambda^{-10} \tag{1}$$

---

[5] Parry Moon, *J. Franklin Inst.*, **230**, 583 (1940), Table III.

and the constants were evaluated so that the approximation would pass through the three points: 0.40 μ, 550; 0.55 μ, 1196; 0.70 μ, 1147 watt m.$^{-2}$ micron^{-1}. The constants were found to be

$$K_0 = 1510.8,$$
$$K_1 = 515,$$
$$K_2 = 0.07915.$$

The curve plotted from Eq. (1) is shown dashed in Fig. 1. Some fairly

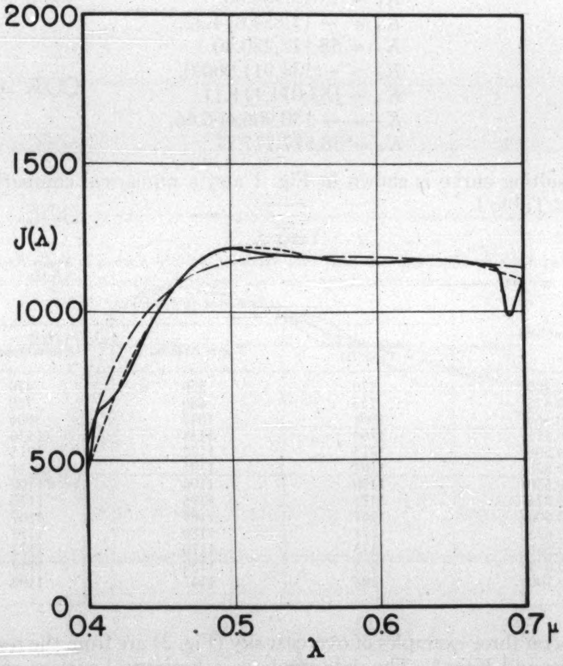

FIG. 1. Direct sunlight. Ordinates are in watt m.$^{-2}$ micron^{-1} incident on a surface that is normal to the sun's rays. The observer is at sea level, the sun is at zenith angle 60°, and the atmosphere contains an average amount of dust and water vapor. ——, Plotted from data in J. Franklin Inst.[5]; — —, Computed, three-term polynomial; - - -, Computed, seven-term polynomial.

large discrepancies are obtained; but these discrepancies are small compared with the large variations obtained in solar spectroradiometric curves because of variations in zenith angle and in the amounts of smoke, dust, and water vapor.

Thus for many purposes, the three-term approximation is sufficiently good. If a closer approximation is desired, however, it can be obtained by employing more terms. A seven-term polynomial illustrates the possibilities. The equation is

$$J(\lambda) = K_0 + K_1\lambda + K_2\lambda^2 + \cdots + K^6\lambda^6. \tag{2}$$

Constants were evaluated by the method discussed previously[3] and were found to be

$$\begin{aligned} K_0 &= 1{,}043{,}968.00, \\ K_1 &= -\ 11{,}854{,}674.32, \\ K_2 &= 55{,}178{,}280.40, \\ K_3 &= -\ 134{,}911{,}000.0, \\ K_4 &= 183{,}071{,}111.11, \\ K_5 &= -\ 130{,}906{,}666.66, \\ K_6 &= 38{,}577{,}777.77. \end{aligned}$$

The resulting curve is shown in Fig. 1 and a numerical comparison is given in Table I.

TABLE I.

Direct Solar Irradiation at Sea Level with Receiving Surface Normal to Sun's Rays.

λ (micron)	$J(\lambda)$ in watt m.$^{-2}$ micron^{-1}		
	Reference 5, Table III	3-term Approx.	7-term Approx.
0.400	470	550	470
0.425	755	880	760
0.450	1006	1047	1006
0.475	1161	1131	1156
0.500	1215	1172	1215
0.525	1194	1191	1214
0.550	1190	1196	1190
0.575	1173	1195	1170
0.600	1167	1189	1167
0.625	1171	1180	1173
0.650	1173	1170	1173
0.675	1155	1159	1149
0.700	1108	1147	1108

Data on three examples of overcast sky (Fig. 2) are from the research of Taylor and Kerr.[6] The data apply to a horizontal surface exposed to the entire sky. Considerable variations were found among the experimental curves, particularly at the shorter wavelengths. A simple expression representing an approximate average of the three might be useful in engineering calculations dealing with overcast skies. The relative values of Taylor and Kerr were put on an absolute basis by using an average datum from the investigation of Kimball and Hand.[7] A three-term polynomial was then passed through the points: 0.40 μ, 248.7; 0.55 μ, 280.9; 0.70 μ, 234.0 watt m.$^{-2}$ micron^{-1}. The heavy curve of Fig. 2 was plotted from the resulting equation:

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[6] A. H. Taylor and G. P. Kerr, *J. Opt. Soc. Am.*, **31**, 3 (1941).

[7] H. H. Kimball and I. F. Hand, *Trans. I.E.S.*, **16**, 255 (1921).

$$J(\lambda) = 83.947 + 42.857\lambda^{-2} - 0.017787\lambda^{-10}. \qquad (3)$$
$$(\text{watt m.}^{-2}\ \text{micron}^{-1})$$

Evidently Eq. (3) represents average conditions for overcast skies to within the variations to be expected from day to day or from hour to hour. Thus, at least until more complete data are available, *Eq.* (*3*) *may be employed in engineering calculations dealing with overcast skies.*

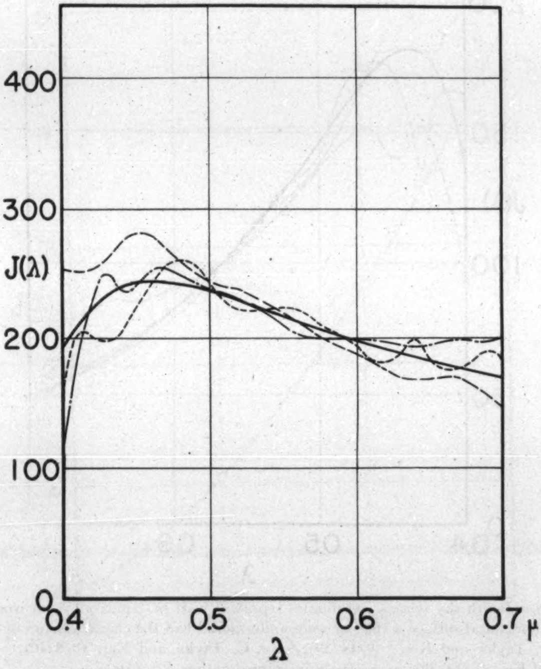

FIG. 2. Average overcast sky. Ordinates are in watt m.$^{-2}$ micron^{-1} incident on an unobstructed horizontal surface. — —, A, Taylor and Kerr[6] (9/13/39); — — —, B, Taylor and Kerr (10/11/39); - - - -, C, Taylor and Kerr (8/27/40); ——, Analytic approximation, Eq. (3).

A similar treatment was employed for blue sky, Fig. 3. The data are from Taylor and Kerr,[6] put on an absolute basis by use of a zenith helios of approximately 7500 blondels (Kimball and Hand,[7] clear sky, sun at 50° zenith angle). The ordinates therefore refer to conditions near the zenith, or they may be regarded as watt $m.^{-2}$ $micron^{-1}$ that would be received on a horizontal plane if the sky were uniform, with helios equal to 7500 blondels. The dotted curves represent data of

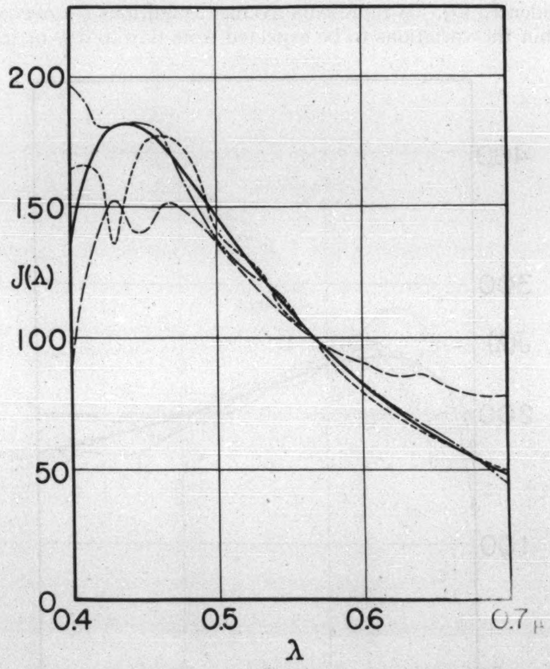

Fig. 3. Zenith sky (clear). Ordinates represent watt $m.^{-2}$ $micron^{-1}$ that would be incident on a horizontal surface if the sky were uniform and had the characteristics of zenith sky. — — —, B, Taylor and Kerr [8] (9/14/39); - - -, C, Taylor and Kerr (8/8/40); — — —, D, Taylor and Kerr (9/14/39); ——, Analytic approximation, Eq. (4).

Taylor and Kerr. The heavy curve shows the analytic representation:

$$J(\lambda) = 2.7031 + 11.0315\lambda^{-4} - 0.031226\lambda^{-10}. \qquad (4)$$

$$(\text{watt m.}^{-2}\ \text{micron}^{-1})$$

In view of the large variations in the experimental data, there seems to be no need to employ more complicated expressions than this simple three-term polynomial.

3. INCANDESCENT LAMPS.

The incandescent lamp is another source whose spectral distribution is often needed in engineering calculations. Data are given by Barnes and Forsythe [8] and by Forsythe and Adams.[9] As representative of the lowest temperatures employed in modern tungsten-filament lamps, consider the 250-watt, 120-volt drying lamp operating at a color temperature of 2530° K. The measurements [9] are indicated by the points of Fig. 4. The ordinates of the graph represent watts m.$^{-2}$ micron^{-1} incident on a surface 1.0 m. from the center of the lamp filament, the receiving surface being parallel to the plane of the filament leads.[9] The curve

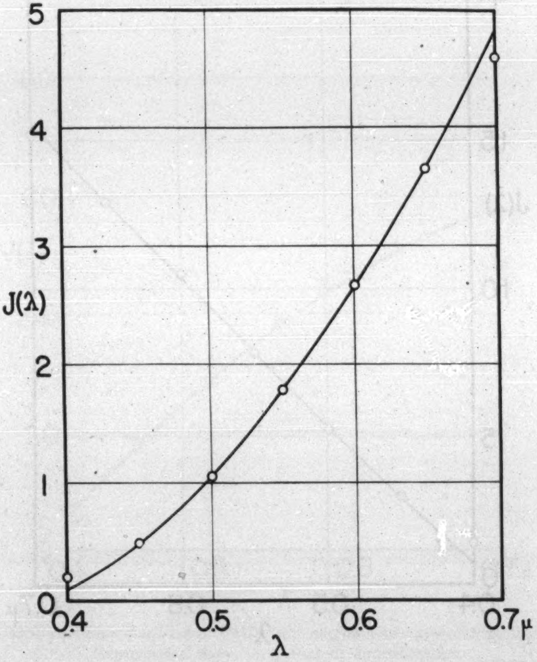

FIG. 4. Spectral distribution for a 250-watt drying lamp. Ordinates represent watt m.$^{-2}$ micron^{-1} incident on a surface 1.0 m. from the lamp. ○, Experimental data of Barnes; [9] ——, Three-term analytic approximation.

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[8] B. T. Barnes and W. E. Forsythe, *J. Opt. Soc. Am.*, **26**, 313 (1936).

[9] W. E. Forsythe and E. Q. Adams, *J. Opt. Soc. Am.*, **35**, 108 (1945).

represents the three-term analytic approximation,

$$J(\lambda) = K_0 + K_1\lambda + K_2\lambda^2, \tag{5}$$

passed through the experimental points at 0.45, 0.55, and 0.65 μ. The routine method of determining the coefficients is explained elsewhere.[4]

Equation (5) applies to the entire range of filament temperatures, just as it applies to Planckian radiation.[4] Figure 5 shows the data for a 500-watt, 115-volt tungsten-filament lamp and its three-term analytic approximation. Figure 6 gives similar information on the 1000-watt Photoflood lamp, whose filament operates not far below the melting point of tungsten. It will be noted from Figs. 4, 5, and 6 that the analytic approximation is in good agreement with the experimental data

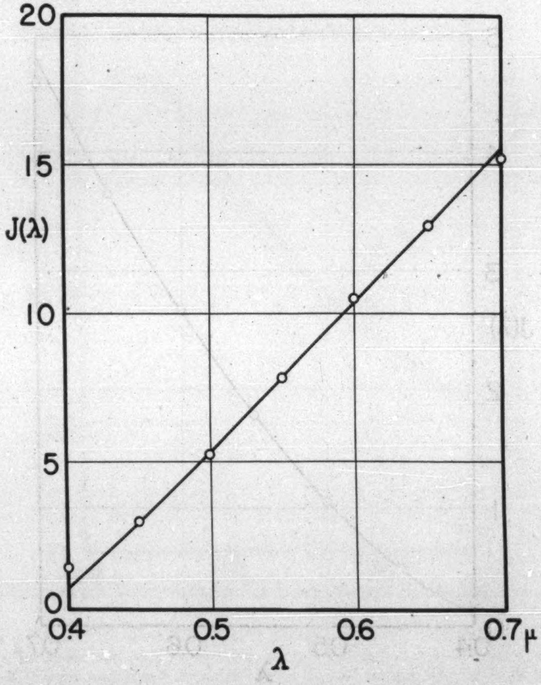

FIG. 5. 500-watt, 115-volt, general-service, gas-filled tungsten lamp. Watt m.$^{-2}$ micron^{-1} on surface 1.0 m. from lamp. ○, Experimental data of Barnes;[9] ——, Three-term analytic approximation.

except at the extreme ends of the graphs. The three-term analytic approximation in the visible region is sufficient to represent lamp data to within the present limits of measurement. If a more exact representation of a particular set of data is desired, however, it is easily obtained by using the seven-term polynomial, as was done with the sunlight curve of Fig. 1.

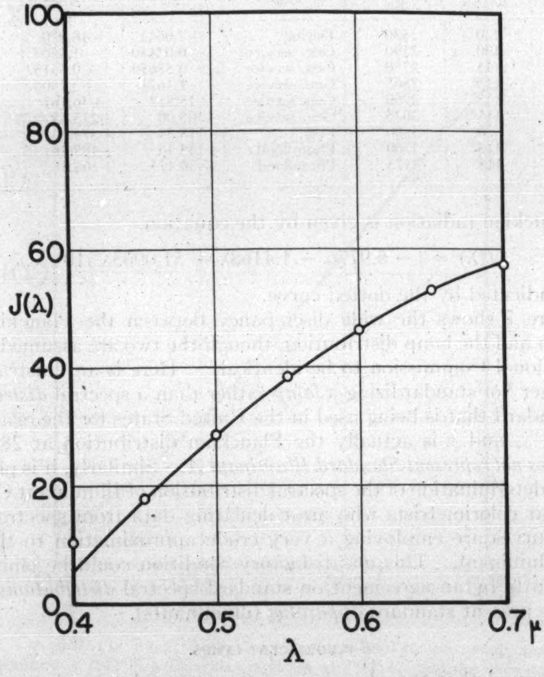

FIG. 6. 1000-watt Photoflood lamp. Watt m.$^{-2}$ micron^{-1} on surface 1.0 m. from lamp. ○, Experimental data; ——, Analytic approximation.

The constants of Eq. (5) were evaluated for a number of incandescent lamps, and the values are listed in Table II. Forsythe and Adams give data also for a tungsten lamp representing Standard Illuminant A. Their data (multiplied by a constant to make the point at 0.60 μ coincide with the Planckian curve) are indicated by the points of Fig. 7. Planckian radiation for 2842° K. (with $C_2 = 14320$) was calculated[2] and is represented by the heavy curve. The three-term approximation to

TABLE II.

Incandescent Lamps.

Based on the data of Forsythe and Adams, *J. Opt. Soc. Am.*, **35**, 108 (1945).

$$J(\lambda) = K_0 + K_1\lambda + K_2\lambda^2$$

(watt m.$^{-2}$ micron^{-1} at 1.0 m. from lamp on a plane that is parallel to the plane containing the filament leads).

P (Watts)	V (Volts)	T_c (°K.)	Type	K_0	K_1	K_2
250	120	2530	Drying	+2.0641	−16.870	+29.750
25	120	2590	Gen. service	+0.03150	−1.3500	+3.2000
40	115	2750	Gen. service	−0.55659	+0.55132	+2.1714
100	115	2865	Gen. service	−2.1672	+3.7506	+4.6389
500	115	2960	Gen. service	−15.822	+36.167	+12.390
1500	115	3025	Gen. service	−105.00	+275.55	−8.2500
900	30	3200	Proj.	−118.34	+375.86	−175.78
1000	115	3360	Photoflood	−143.46	+485.38	−281.88
250	115	3475	Photoflood	−30.475	+106.38	−66.521

this Planckian radiation is given by the equation,

$$J(\lambda) = (-6.9776 - 1.4168\lambda + 51.0905\lambda^2)10^5,$$

and is indicated by the dotted curve.

Figure 7 shows the wide discrepancy between the Planckian distribution and the lamp distribution, though the two are assumed by the International Commission to be identical.[10] Here is an illustration of the danger [2] of standardizing a *lamp* rather than a spectral *distribution.* The standard that is being used in the United States for the evaluation of ρ_l, τ_l, x, and y is actually the Planckian distribution at 2842° K., *which does not represent Standard Illuminant A*. Similarly, it is probable that a redetermination of the spectral distribution of Illuminant C would show that colorimetrists who are calculating data from spectrophotometric curves are employing a very crude approximation to the true C.I.E. illuminant. This unsatisfactory condition could be eliminated permanently by an agreement on standard spectral *distributions* to replace the present standard *apparatus* (illuminants).

4. FLUORESCENT LAMPS.

It is found that the phosphors of fluorescent lamps give radiation curves that are representable by Pearson functions. Besides the continuous spectrum produced by the phosphors, fluorescent lamps also emit the mercury spectrum.

The continuous spectrum will be represented by equations of the form,

$$J(\lambda) = A' \frac{\exp(-q'/\lambda)}{\lambda^{p'}}, \qquad (6)$$

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[10] Commission Internationale de l'Eclairage, "Compte Rendu des Séances 1931," Cambridge University Press, 1932, pp. 19, 22.

where A', p', and q' are constants. This is the same form of equation that was used in representing trichromatic weighting functions,[1,2] where the constants were determined by the method of least squares. For the present purpose, a simpler method of evaluating A', p', and q' is desirable.

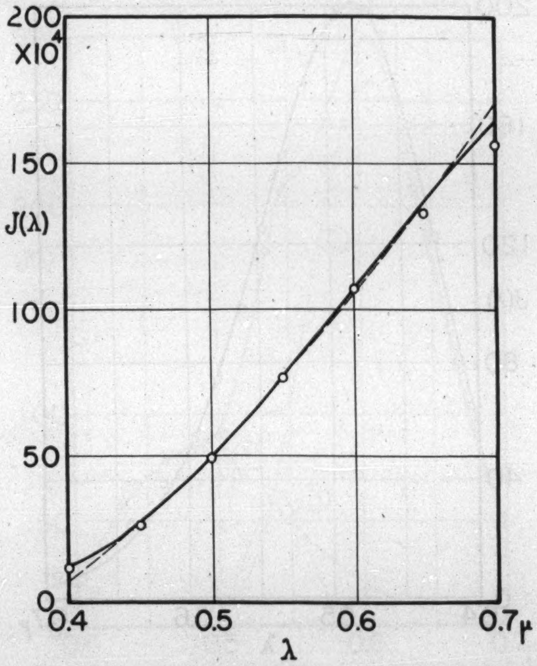

FIG. 7. Comparison of Standard Illuminant "A" (gas-filled tungsten lamp operating at a color temperature of 2842° K.), Planckian radiation at 2842° K., and the three-term polynomial approximation to Planckian radiation. The ordinates represent watt micron^{-1} emitted per sq. m. of Planckian-radiator opening. The lamp data have been multiplied by a constant to obtain coincidence with the Planckian data at $\lambda = 0.60\ \mu$. ○, Tungsten lamp at 2842° K. color temperature, experimental results of Barnes; ——, Planckian radiation at 2842° K.; — —, Polynomial approximation to Planck curve.

Equation (6) may be written

$$\log J(\lambda) = \log A' + p' \log (1/\lambda) - 0.4342945q'/\lambda. \tag{6a}$$

Suppose that the maximum of the experimental curve occurs at wavelength λ_m. Select one other wavelength λ_1 and make Eq. (6a) coincide with the experimental values at these two wavelengths. Then

$$\begin{cases} \log J(\lambda_m) = \log A' + p' \log (1/\lambda_m) - 0.4342945q'/\lambda_m, \\ \log J(\lambda_1) = \log A' + p' \log (1/\lambda_1) - 0.4342945q'/\lambda_1. \end{cases}$$

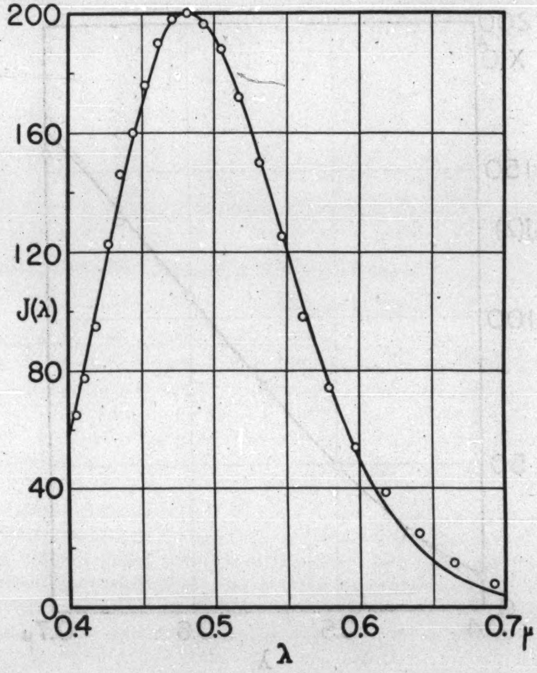

FIG. 8. Spectral distribution curve for a fluorescent lamp, phosphor = magnesium tungstate. The ordinates represent average watts per square meter of phosphor, per micron wavelength band. The experimental data (relative values) were normalized so that the curve could be used directly in the daylight lamp (Fig. 10). ○, Experimental data of Barnes; ——, Analytic approximations, Eq. (6).

But the peak of the curve occurs at $\lambda_m = q'/p'$, so

$$\begin{cases} p' = \dfrac{\log [J(\lambda_m)/J(\lambda_1)]}{0.4342945(\lambda_m/\lambda_1 - 1) - \log (\lambda_m/\lambda_1)}, & (7) \\ q' = \lambda_m p', \\ \log A' = \log J(\lambda_m) + p(0.4342945 - \log (1/\lambda_m)). & (9) \end{cases}$$

If $\lambda_1 > \lambda_m$, it is convenient to replace Eq. (7) by its equivalent,

$$p' = \frac{\log [J(\lambda_m)/J(\lambda_1)]}{\log (\lambda_1/\lambda_m) - 0.4342945(1 - \lambda_m/\lambda_1)} \tag{7a}$$

These equations allow the evaluation of the constants of Eq. (6).

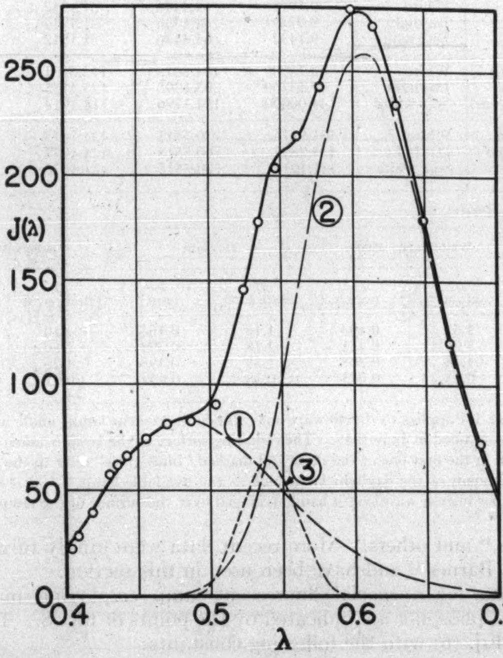

FIG. 9. Spectral distribution for a white fluorescent lamp (40 watts). Phosphors = magnesium tungstate and zinc beryllium silicate. Ordinates are in watt m.$^{-2}$ micron^{-1} at the lamp surface. ○, Experimental results of Barnes; ——, Analytic approximation; 1, Distribution curve for magnesium tungstate; 2, Distribution curve for zinc beryllium silicate, principal part of analytic approximation; 3, Correction curve, zinc beryllium silicate.

Information on the spectral distributions of fluorescent lamps is given by Inman,[11] Weitz and Cissell,[12] Oetting and Amick,[13] Forsythe, Barnes,

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[11] G. E. Inman, *Trans. I.E.S.*, **34**, 65 (1939).

[12] C. E. Weitz and R. F. Cissell, *Trans. I.E.S.*, **34**, 495 (1939).

[13] R. L. Oetting and C. L. Amick, *Illum. Eng.*, **36**, 1369 (1941); C. L. Amick, "Fluorescent Lighting Manual," McGraw-Hill Book Co., New York, 1942.

TABLE III.

Fluorescent Lamps.

(a) *Continuous Spectra*

$$\log J(\lambda) = \log A' + p' \log (1/\lambda) - 0.4342945q'/\lambda \text{ (watt m.}^{-2}\text{ micron}^{-1}\text{)}.$$

Material	Lamp	log A'	p'	q'	$0.4342945q'$
Magnesium tungstate	Blue-white	9.9789	64.4446	31.1912	13.5462
	White	9.5978	64.4446	31.1912	13.5462
	Daylight	9.9789	64.4446	31.1912	13.5462
	Soft white	9.7432	64.4446	31.1912	13.5462
Zinc beryllium silicate (principal term)	White	42.73800	184.8078	112.3631	48.7987
	Daylight	47.53200	203.4398	125.1155	54.3370
	Soft white	44.00000	184.1796	114.1914	49.5927
Zinc beryllium silicate (correction term)	White	131.9176	780.5515	421.4978	183.0542
	Daylight	131.2847	780.5515	421.4978	183.0542
	Soft white	131.1014	780.5515	421.4978	183.0542

(b) *Line Spectra*

λ (micron)	White Lamp (3500°)		Daylight		Soft White	
	D_r (watt m.$^{-2}$)	F_r (watt)	D_r (watt m.$^{-2}$)	F_r (watt)	D_r (watt m.$^{-2}$)	F_r (watt)
0.405	1.37	0.193	1.16	0.164	1.10	0.155
0.436	3.06	0.431	2.78	0.392	2.56	0.361
0.546	1.48	0.208	1.38	0.195	1.26	0.178
0.578	0.53	0.075	0.38	0.054	0.41	0.058

Note: Table III applies to the 40-watt hot-cathode fluorescent lamps made in the U.S.A. The data were obtained in June 1945. The radiating surface of the lamp is taken as 0.141 m^2. The data given in the first line of the table and marked "blue-white" refer to the magnesium-tungstate component of the daylight lamp, not to any available lamp. $J(\lambda)$ is in watts per square meter, per micron wavelength band, averaged over the surface of the lamp.

and Adams,[14] and others. More recent data were kindly furnished by Amick and Barnes [15] and have been used in this section.

The data for a special fluorescent lamp employing magnesium tungstate as phosphor are indicated by the points of Fig. 8. The curve represents Eq. (6) with the following constants:

$$\log A' = 9.9789,$$
$$p' = 64.4446,$$
$$q' = 31.1912.$$

The "white" fluorescent lamp employs a mixture of magnesium tungstate and zinc beryllium silicate. The experimental data are indicated by the points of Fig. 9. The part of the curve from 0.40 to 0.48 μ is fitted by the magnesium tungstate curve (Fig. 8) with all ordinates multiplied by 0.297. Subtraction of this Pearson curve from the data of Fig. 9 must leave the distribution produced by the zinc beryllium silicate. It is found that the latter cannot be represented by a single Pearson function, though it can be represented as the sum of two such functions. The constants are listed in Table III. The resulting curve is plotted in Fig. 9 and is found to be in good agreement with the experimental data.

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[14] Forsythe, Barnes, and Adams, *Denison Univ. Bulletin*, **36**, 13 (1941).

[15] The authors gratefully acknowledge the help of Mr. C. L. Amick, Nela Park Engineering Department of the General Electric Company, Dr. B. T. Barnes, Lamp Development Laboratory of the General Electric Co., and Dr. E. M. Lowry, Sylvania Electric Products, all of whom furnished information on fluorescent lamps.

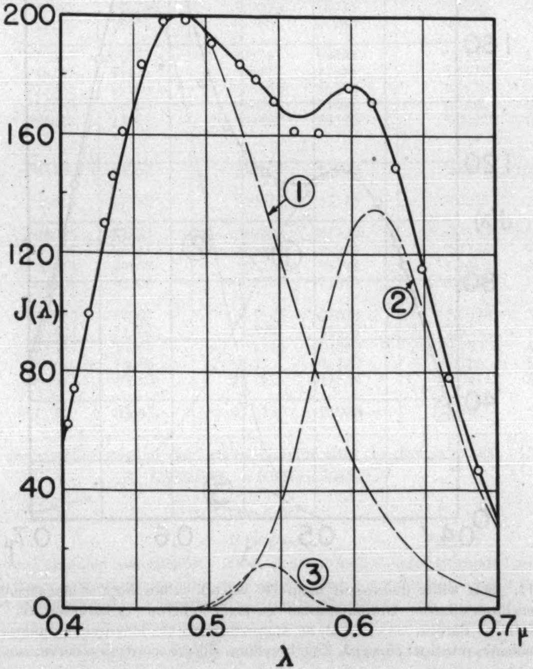

FIG. 10. Daylight fluorescent lamp (40 watts). Phosphors = magnesium tungstate and zinc beryllium silicate. Ordinates are in watts m.$^{-2}$ micron^{-1} at lamp surface. ○, Experimental results of Barnes; ——, Analytic approximation; 1, Magnesium tungstate; 2, Zinc beryllium silicate, principal curve; 3, Zinc beryllium silicate, correction curve.

The "soft white" and the "daylight" fluorescent lamps are made with the same two materials, but the zinc beryllium silicate is prepared in different ways so that the long-wavelength peaks come at different wavelengths. Data on the constants are given in Table III and curves are plotted in Figs. 10 and 11.

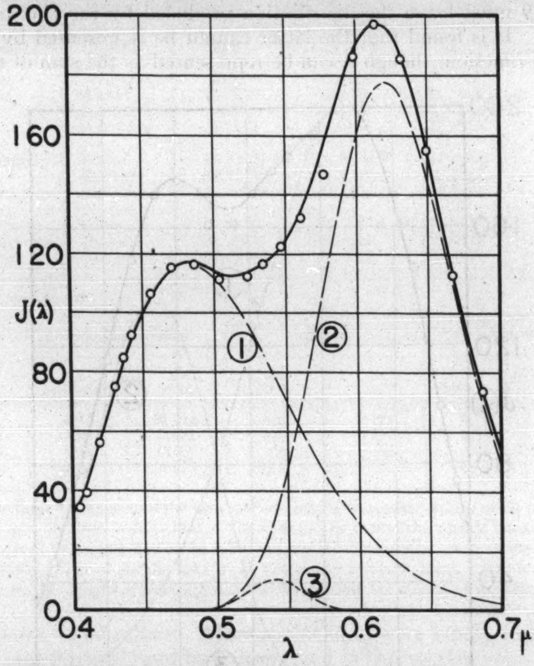

FIG. 11. Soft white fluorescent lamp (40 watts). Phosphors = magnesium tungstate and zinc beryllium silicate. Ordinates are in watts m.$^{-2}$ micron^{-1} at lamp surface. ○, Experimental results of Barnes; ——, Analytic approximation; 1, Magnesium tungstate; 2, Zinc beryllium silicate, principal curve; 3, Zinc beryllium silicate, correction curve.

5. INTEGRATION.

Integration can be performed, using the previous analytic representations. For daylight, power functions are used, and the weighted integrations are performed very easily [4] by the use of Table IV. Values of (L/K_m) are taken from the table and are multiplied by the coefficients K_m of the polynomials, Eqs. (1) to (4). The integrals are

$$L = \int_0^\infty w(\lambda)J(\lambda)d\lambda = \sum_m (L/K_m)K_m, \tag{10}$$

where L is a generic designation for X, Y, or Z. Values obtained in this way are listed in Table V and are compared with results obtained

TABLE IV.

Coefficients for Integration of Polynomial Approximation, Light Sources

m	X/K_m	Y/K_m	Z/K_m
0	$^{0}10666$ 57240	$^{0}10685$ 35175	$^{0}10677$ 25053
1	$^{1}60921$ 26156	$^{1}59773$ 82336	$^{1}48473$ 18484
2	$^{1}35226$ 72563	$^{1}33616$ 34585	$^{1}22066$ 86373
3	$^{1}20585$ 29574	$^{1}19006$ 76935	$^{1}10073$ 48896
4	$^{1}12138$ 18444	$^{1}10804$ 00986	$^{2}46112$ 93726
5	$^{2}72128$ 04822	$^{2}61741$ 99087	$^{2}21167$ 64643
6	$^{2}43147$ 45424	$^{2}35472$ 77575	$^{3}97438$ 95977
7	$^{2}25962$ 45164	$^{2}20489$ 36516	$^{3}44978$ 64475
8	$^{2}15703$ 36300	$^{2}11898$ 18699	$^{3}20820$ 78752
9	$^{3}95428$ 23493	$^{3}69462$ 88390	$^{4}96651$ 46038
10	$^{3}67101$ 94957	$^{3}40770$ 52600	$^{4}44992$ 86102
0	$^{0}10666$ 57240	$^{0}10685$ 35175	$^{0}10677$ 25053
−1	$^{0}18945$ 52333	$^{0}19203$ 57729	$^{0}23583$ 64653
−2	$^{0}34207$ 93241	$^{0}34696$ 72926	$^{0}52233$ 94835
−3	$^{0}62918$ 49167	$^{0}63023$ 83522	1.1600 63798
−4	1.1809 92005	1.1508 72075	2.5834 18571
−5	2.2652 57902	2.1127 61624	5.7688 38040
−6	4.4431 71306	3.8991 48094	12.916 93227
−7	8.9117 26706	7.2340 01869	29.000 44752
−8	18.261 13996	13.491 81539	65.286 16073
−9	38.177 40910	25.295 17554	147.36 87925
−10	81.275 83667	47.673 01938	333.54 52442

NOTE.—Superscripts refer to the number of zeros after the decimal point. Thus $^{2}7212804822 = 0.007212804822$.

TABLE V.

Daylight.

Integrations obtained analytically. X, Y, Z are in m.k.s. units, evaluated with respect to the trichromatic weighting functions, on a surface that is normal to the sun's rays (in the case of direct sunlight) and on a horizontal surface (in the other cases).

Condition	X (könig m.$^{-2}$)	Y (young m.$^{-2}$)	$647.8Y$ (lumens/m.^{2})	Z (priest m.$^{-2}$)	x	y
Direct Sunlight						
3-term polynomial	123.34	126.88	8.219×10^{4}	109.95	0.3425	0.3523
7-term polynomial	121.16	126.70	8.208	106.23	0.3422	0.3578
J. Franklin Inst.			8.178		0.3431	0.3567
Taylor and Kerr					0.336	0.350
Overcast Sky						
3-term polynomial	22.169	22.992	1.49	25.416	0.3141	0.3258
Taylor and Kerr					0.313	0.328
Blue Sky						
3-term polynomial	10.779	11.496	0.74	18.372	0.2652	0.2828
Taylor and Kerr					0.263	0.279

Note: The values marked "Taylor and Kerr" are not strictly comparable with the analytic integrations, since the averages they use are different from ours.

by Taylor and Kerr.[9] Similar analytic integrations for incandescent lamps are summarized in Table VI.

TABLE VI.

Incandescent Lamps.

Results of analytic integration of the three-term expressions of Table II. The values of X, Y, Z are in m.k.s. units, incident on a surface 1.0 m. from the lamp, evaluated with respect to the trichromatic weighting functions $w(\lambda)$.

P (watts)	Type	X (könig m.$^{-2}$)	Y (young m.$^{-2}$)	Z (priest m.$^{-2}$)	x	y
250	Drying	0.2404	0.2123	0.0591	0.4698	0.4147
25	Gen. service	0.0338	0.0302	0.0085	0.4660	0.4164
40	Gen. service	0.0507	0.0465	0.0152	0.4512	0.4135
100	Gen. service	0.1607	0.1486	0.0528	0.4439	0.4103
500	Gen. service	0.9521	0.8877	0.3372	0.4374	0.4078
1500	Gen. service	5.2963	4.9737	1.9636	0.4329	0.4066
900	Projection	4.0829	3.9125	1.7048	0.4209	0.4033
1000	Photoflood	4.3380	4.2080	1.9901	0.4117	0.3994
250	Photoflood	0.8868	0.8662	0.4348	0.4054	0.3959

When $J(\lambda)$ is in watt m.$^{-2}$ micron^{-1} and the weighting function $w(\lambda)$ is $\bar{y}(\lambda)$ of the C.I.E. trichromatic system, then Y obtained from Eq. (10) is not in lumens per square meter but is in *youngs* per square meter.[16] The conversion factor between the lumen and the young is still somewhat in doubt,[17] but in this paper the factor will be taken as 635 old lumens per young or 647.8 new lumens per young. Corresponding to the unit for Y are the units for X and Z, which will be called respectively the *könig* per square meter and the *priest* per square meter.[16]

The calculated results for incandescent lamps are shown graphically in Fig. 12. The curve is the Planckian locus, calculated by means of the analytic weighting functions.[1] The black dots represent Planckian radiations calculated by numerical integration using the C.I.E. tabulated weighting factors. The discrepancy is small,[2] particularly at the higher temperatures. The white dots are for incandescent lamps. Note that the radiation from all these lamps is decidedly greener than Planckian radiation.

For the Pearson functions used in representing the spectroradiometric curves for fluorescent lamps, integration is performed by use of the equation,

$$\log L = \log A + \log A' + \log \Gamma(p + p' - 1) \\ - (p + p' - 1) \log (q + q'). \quad (11)$$

The unprimed letters refer to the analytic weighting functions [2] and are

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[16] Parry Moon and D. E. Spencer, "Units in the Trichromatic System," *J. Opt. Soc. Am.*, **36**, 120 (1946).

[17] D. L. MacAdam, "Note concerning the Maximum Luminous Efficiency of Radiant Energy," *J. Opt. Soc. Am.*, **35**, 615 (1945).

TABLE VII.

Values of the Constants in Eq. (4).

	$\bar{z}(\lambda)$	$\bar{x}_B(\lambda)$	$\bar{x}_C(\lambda)$	$\bar{y}_A(\lambda)$	$\bar{y}_B(\lambda)$	$\bar{y}_D(\lambda)$
p	365.33880	500.00000	336.03850	182.19050	3.20500	1013.50
q	164.95060	237.50000	199.10540	100.93700	0.12820	679.0450
$\log q$	2.21735 38996	2.37566 36140	2.29908 30387	2.00405 03928	9.10788 80252–10	2.83189 85558
$\log A$	32.75260	54.51150	69.59480	32.41380	5.02860–10	262.03010

$\bar{x}_A(\lambda) = \bar{z}(\lambda)/4.570,$ $\bar{y}_C(\lambda) = 0.36678\,13725\bar{x}_C(\lambda).$

Note: Final zeros indicate that the values are exact by definition to any number of decimals.

obtained from Table VII. The primed letters refer to the lamp radiation (Table III). The gamma function is obtained from tables [18] or is

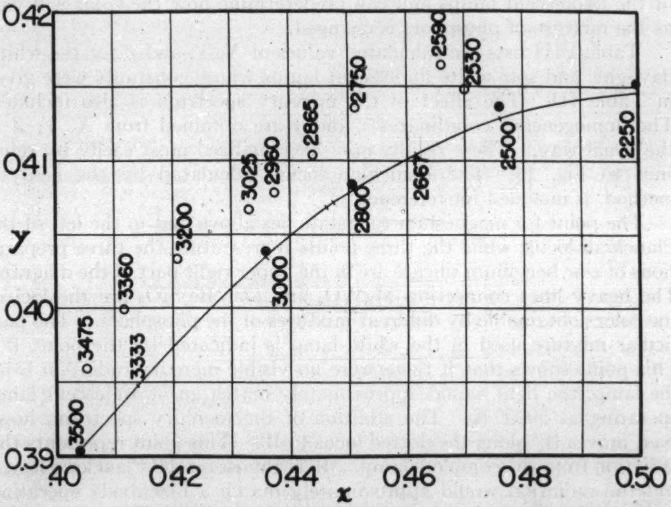

FIG. 12. Homogeneous coordinates of the radiation from incandescent lamps. ○, Incandescent lamps, x and y, calculated by the analytic method; numbers are color temperatures as given by Forsythe and Adams; ——, Planckian locus, calculated by the analytic method; ●, Planckian radiation, numerical integration of C.I.E. data.

calculated by Stirling's formula,

$$\log \Gamma(x) = 0.39908\,993 + (x - \tfrac{1}{2}) \log (x - 1) - 0.4342945(x - 1) + \log \left[1 + \frac{1}{12(x-1)} + \frac{1}{288(x-1)^2} - \cdots \right]. \quad (12)$$

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[18] J. Peters, "Zehnstellige Logarithmen" (Berlin, 1922), Vol. I, Table 6.

For the magnesium tungstate curve, for example, $w(\lambda) = \bar{z}(\lambda)$,

$$\begin{aligned} (p + p' - 1) &= 428.78339, \\ (q + q') &= 196.14178, \\ \log \Gamma(p + p' - 1) &= 941.52471, \\ (p + p' - 1) \log (q + q') &= 983.01598, \\ \log A &= 32.75260, \\ \log A' &= 9.9789. \end{aligned}$$

Thus, from Eq. (11),

$$Z = 17.3892 \text{ könig m}^{-2}.$$

In this way one can compute the trichromatic coordinates for any of the fluorescent lamps and can predetermine how the color will vary as the mixture of phosphors is changed.

Table VIII lists the calculated values of X, Y, and Z for the white, daylight, and soft white fluorescent lamps whose constants were given in Table III. The effect of the mercury spectrum is also included. The homogeneous coordinates x and y are obtained from X, Y, Z in the usual way. These results may be visualized most easily by reference to Fig. 13. The Planckian locus, calculated by the analytic method, is included for reference.

The point for magnesium tungstate lies above and to the left of the Planckian locus, while the three points representing the three preparations of zinc beryllium silicate are in the upper right part of the diagram. The heavy lines connecting $MgWO_4$ and $(Zn, Be)SiO_3$ are the loci of the colors obtainable by different mixtures of the phosphors. The particular mixture used in the white lamp is indicated by the point W'. This point shows that if there were no visible mercury radiation from the lamp, the light would approximately match an incandescent lamp operating at 3000° K. The addition of the mercury spectrum, however, moves W' along the dotted locus to W. This point represents the radiation from the complete lamp. W is almost on the Planckian locus, and the radiation would approximately match a blackbody operating at 3500° K. Similarly, the particular mixture of phosphors used in the daylight lamp gives point D', while inclusion of the mercury spectrum gives D. It is interesting to note that the whole Planckian locus from 1000° to ∞ can be covered by proper mixtures of two phosphors, plus direct mercury radiation.

6. REFLECTED RADIATION.

The colorimetrist must be able also to specify the trichromatic coordinates of the radiation that is reflected from a surface or transmitted through a filter. In particular, he may wish to study the color distortion produced by the mercury spectrum of a fluorescent lamp. All such questions can be handled readily by the analytic method.

TABLE VIII.

Calculated Characteristics of 40-Watt Fluorescent Lamps.

3500° White Lamp	X (könig m.$^{-2}$)	Y (young m.$^{-2}$)	Z (priest m.$^{-2}$)
Magnesium tungstate	3.4336	4.8504	7.2304
Zinc beryllium silicate	18.9644	16.5292	0.1823
Mercury spectrum	1.9902	1.9942	4.6060
Total	24.3882	23.3738	12.0187

Lumens per square meter of glass surface = 15,142
Total lumens = 2135

	x	y
$MgWO_4$ only	0.2213	0.3126
(Zn, Be) SiO_3 only	0.5316	0.4633
Hg only	0.2317	0.2321
Continuous spectrum only	0.4375	0.4177
Total lamp	0.4080	0.3910

Daylight Lamp	X (könig m.$^{-2}$)	Y (young m.$^{-2}$)	Z (priest m.$^{-2}$)
Magnesium tungstate	8.2580	11.6650	17.3892
Zinc beryllium silicate	9.0280	6.7622	0.0424
Mercury spectrum	1.7200	1.7544	4.1808
Total	19.0060	20.1816	21.6124

Lumens per square meter of glass surface = 13,074
Total lumens = 1843

	x	y
$MgWO_4$	0.2213	0.3126
(Zn, Be) SiO_3 only	0.5702	0.4271
Continuous spectrum only	0.3253	0.3467
Total lamp	0.3126	0.3319

Soft White Lamp	X (könig m.$^{-2}$)	Y (young m.$^{-2}$)	Z (priest m.$^{-2}$)
Magnesium tungstate	4.7991	6.7792	10.1057
Zinc beryllium silicate	11.5288	8.3150	0.0437
Mercury spectrum	1.6446	1.6599	3.8581
Total	17.9725	16.7541	14.0075

Lumens per square meter of glass surface = 10,853
Total lumens = 1530

	x	y
$MgWO_4$ only	0.2213	0.3126
(Zn, Be) SiO_3 only	0.5797	0.4181
Continuous spectrum only	0.3928	0.3631
Total lamp	0.3688	0.3438

Note: X, Y, and Z are in m.k.s. units per unit area of glass surface. The radiating surface for the 40-watt lamp is taken as 0.141 m^2. The lumens are "new" lumens.

As an example, consider the reflected radiation from a sample of stained oak (No. 386 of a previous publication [19]) when illuminated by a white fluorescent lamp. On the assumption that the reflectance of the MgO_2 comparison surface was unity, the spectral reflectance of this sample is

$$\rho(\lambda) = -0.0090 + 1.128\lambda^3. \tag{13}$$

The reflected mercury radiation is evaluated by summation, using the data of Table III. The continuous spectrum is handled in two parts, one for the constant term of Eq. (13) and the other for the λ^3-term. The constant term is obtained by merely multiplying X, Y, and Z for the white lamp (Table VIII) by -0.0090, giving

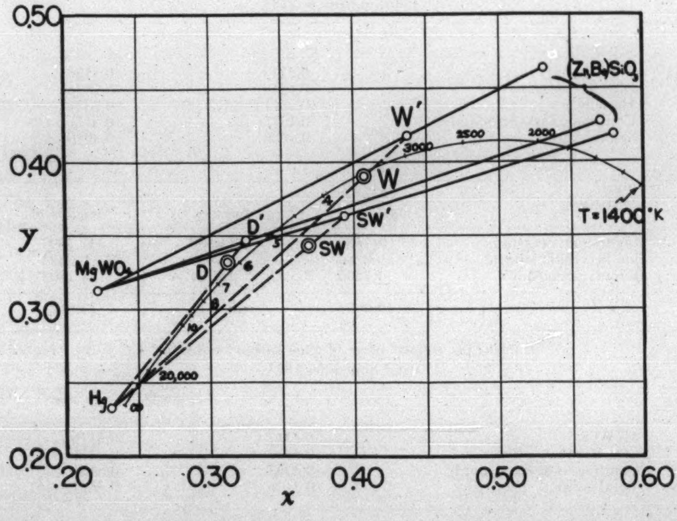

FIG. 13. Homogeneous coordinates of the radiation from 40-watt fluorescent lamps. Calculations by the analytic method. W = white lamp, complete; W' = white lamp with mercury spectrum omitted; D = daylight lamp; D' = daylight lamp, mercury spectrum omitted; SW = soft white lamp; SW' = soft white lamp, mercury spectrum omitted; ——, Planckian locus, calculated by analytic method; Hg = mercury spectrum alone; $MgWO_4$ = magnesium tungstate radiation alone; (Zn, Be)Si_3 = zinc beryllium silicate radiation alone.

$$X = -0.21949, \quad Y = -0.21036, \quad Z = -0.10817.$$

The λ^3-term is calculated by a modification of Eq. (11), which for

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[19] Parry Moon, "Reflection Factors of Floor Materials," *J. Opt. Soc. Am.*, **32**, 238 (1942).

$m = 3$ is

$$\log L = \log A + \log A' + \log 1.128 + \log \Gamma(p + p' - 4) - (p + p' - 4) \log (q + q'). \quad (14)$$

The calculated results are given in Table IX.

TABLE IX.

Reflected Radiation from Oak Under Different Illuminants.

$$\frac{\rho(\lambda)}{\rho_0} = -0.0090 + 1.128\lambda^2.$$

	Hg Component of White Fluorescent	Continuous Component of White Fluorescent	White Fluorescent	Planckian Radiator, 2842°	Illuminant C
X	0.27845	5.0209	5.2994		
Y	0.36352	4.2978	4.6613		
Z	0.38434	0.75211	1.13645		
x	0.27131	0.49856	0.47754	0.5146	0.3968
y	0.35420	0.42676	0.42004	0.4114	0.3749
ρ/ρ_0	0.18229	0.20102	0.19942	0.2043	0.183

In this, as in the previous examples, the purpose has been more to outline the procedure and to indicate the feasibility of the analytic method than to present practical data. With fluorescent lamps, in particular, engineered changes are rather frequent; and inadvertent changes, caused by accidental variations in the purity of the phosphors, are not without their effect. The data used in this paper are supposed to represent the product of one manufacturer as of June 1945, but there is no guarantee that they apply exactly under any other conditions.

7. SUMMARY.

The paper has indicated how analytic methods can be applied to spectroradiometric curves for light sources. Typical curves for direct sunlight, overcast sky, and clear sky are represented by simple three-term polynomials. Polynomials of this kind are employed also in representing incandescent-lamp data over the entire temperature range used in modern tungsten lamps. Fluorescent lamps are handled by use of Pearson functions.

In all cases, the analytic representation agrees with the data to within the experimental variation to be expected in practice. Thus the analytic method gives a simple and compact way of specifying daylight and the radiation from lamps. Differentiation, integration, and other mathematical operations are easily performed. Of particular advantage is the analytic calculation of X, Y, Z, x, y from spectroradiometric curves.

The ideal way of employing the analytic method throughout photometry and colorimetry would be accomplished if international agreement could be reached on mathematical expressions for the three weighting functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. Equations based on several years' study have been suggested.[1,2] These or similar equations would replace the present clumsy method of specifying values of $\bar{x}$, $\bar{y}$, $\bar{z}$ at discrete intervals. They would also allow the calculation, by analytic integration of the trichromatic coordinates, correct by definition to any desired number of decimal places. Failing such international agreement, the investigator may still use the new method, knowing that in the great majority of practical cases it yields results that differ by less than the experimental variation from those obtained by traditional methods.

ON THE SO-CALLED PRINCIPLE OF LEAST WORK METHOD, II.

BY

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INTRODUCTION.

In the previous paper under the same title [1] the Author put forward a proposition for the definition of the So-Called Principle of Least Work Method, as it is named by other writers. In the first part of the present note the Author points out the relation between his proposition and the existing interpretations of the Theorem of Minimum Potential Energy. The relation between the Theorem of Least Work and the Theorem of Minimum Potential Energy has been shown in more than one way by several authors. Hence, a certain similarity and even an identity in the interpretations of both theorems may be expected. In the second part of this note the Author discusses the ways of solution of practical problems by the application of the Theorem of Least Work. Two main cases must be distinguished: concentrated, singular forces and continuously distributed stresses. The first case is trivial. The second case may be attacked in two ways. The first is to choose *a priori* a certain curve or surface representing the approximate stress distribution with certain unknown parameters to be determined by the application of the Theorem of Least Work. The second is the application of the methods of the calculus of variations. Certain earlier works on this subject are mentioned (Courant, Hilbert, Pöschl).

I. The Author previously put forward the following proposition: "When applying Castigliano's theorem of least work to specific problems, we must be assured *a priori* that the redundant generalized forces or unknown stress distributions are expressed by functions among which there is one and only one function which gives a smaller value to the total strain energy than any other gives having first taken the given values on the boundary." The Author would like to point out a complete analogy and even an identity of the above proposition with the forms interpreting the Theorem of Minimum Potential Energy of a system. This must be true, since there exists a close relationship between the Theorem of Least Work and the Theorem of Minimum Potential Energy for structures in which the members obey a linear stress-strain law (Hooke's Law). Such structures are usually subjects of investigations. This relationship has been shown in several ways. It has been shown by Williams in an elementary way.[2]

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[1] M. Z. Krzywoblocki, "On the So-Called Principle of Least Work Method," *Journal of the Franklin Institute*, March, 1947, p. 187. The two diagrams, referring to the appendix attached to the previous paper, not included previously, are included in the present paper.

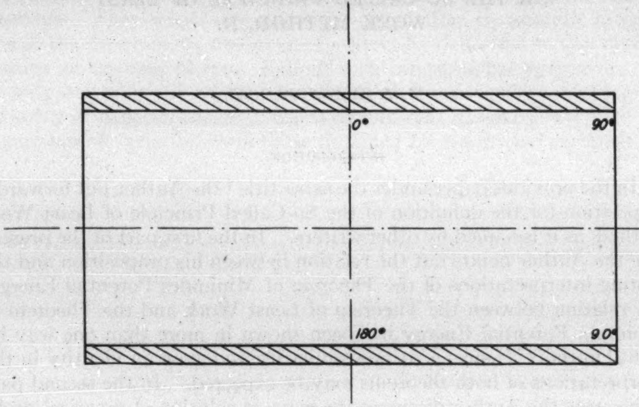

FIG. 1.

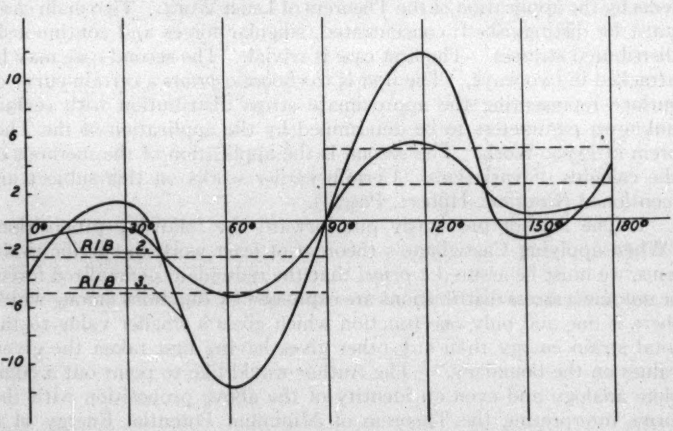

FIG. 2. NORMAL STRESSES.

As a concise statement of the relation between the two theorems it may be said according to Williams that, in applying the Theorem of

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[2] D. Williams, "The Relations between the Energy Theorems Applicable in Structural Theory," *Philosophical Magazine*, Vol. 26, Series 7, November, 1938, p. 618.

Least Work to trusses, say, equilibrium is assumed at each node and the satisfaction of the equations obtained by differentiating the internal energy with respect to the unknown loads in the redundant members ensures compatibility of displacement; whereas in applying the Theorem of Minimum Potential Energy, compatibility of displacement is assumed and the satisfaction of the equations obtained by differentiating the potential energy with respect to the displacements ensures equilibrium at each node.

The equivalence of the two variational forms: one expressing the equilibrium state of a structural system by the principle of minimum potential energy and another expressing the same fact by the principle of minimum strain energy, may be easily shown by use of the transformation methods of the calculus of variations (R. Courant, D. Hilbert, "Methoden der Mathematischen Physik," Berlin, Julius Springer, 1931, Vol. I, p. 230). Courant recommends the application of Friedrich's transformation in this case. Both forms are equivalent from variation calculus standpoint and the meaning of this equivalence lies theoretically in the duality of the variational problems. The correct interpretation of the Theorem of Minimum Potential Energy is very well known: it is necessary to select from the set of all functions satisfying the boundary conditions those functions that make the potential energy a minimum (I. S. Sokolnikoff, "Mathematical Theory of Elasticity," McGraw-Hill Co., 1946, p. 281). A complete analogy and even an identity between the two forms, the one proposed by the Author for the So-Called Principle of Least Work and that one for the Theorem of Minimum Potential Energy, are quite visible.

II. In considering the application of the Theorem of Least Work to the solution of practical problems, two main cases must be distinguished. The separation into these two cases was already being made by Pöschl.[3] (Theodor Pöschl, "Über die Minimalprinzipe der Elastizitätstheorie," *Der Bauingenieur*, Vol. 17, No. 17/18, Mai, 1936, p. 160 to 164. See also discussion of this paper by O. Domke, *Der Bauingenieur*, Vol. 17 No. 41/42, 1936, p. 459.)

The first case is when in the system considered we deal only with the concentrated, separate forces in the members of the structure. In this case the total strain energy of the system is given by a quadratic form which is always a positive definite one, therefore the extremum sought must be a minimum.[4] In this case the solution is given by equating to zero the first partial derivatives of the total strain energy with respect to the redundant forces. Pöschl calls this case "Castigliano's Principle."

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[3] In the Author's previous paper (*loc. cit.*) this distinction is self evident from the exposition of the definition of the So-Called Principle of Least Work Method.

[4] The Author cited in his previous paper various attempts (found in several textbooks) to prove the existence of a minimum by pointing out that the second partial derivatives taken separately with respect to X, Y, Z, etc. are positive. This condition is necessary but not yet sufficient. The fact that the quadratic form is a positive definite is the only proof of existence of a minimum. See: Theodor Pöschl, "Elementare Festigkeitslehre," Berlin, Julius Springer, 1936, p. 173.

Here it may be mentioned that Williams (*loc. cit.*) expresses the opinion that in all highly redundant structures, however, there is little doubt that use of the Theorem of Minimum Potential Energy is more convenient.

The second case occurs when in the system there are continuous distributed stresses. The total strain energy is expressed in this case by single, double or triple integral forms and the equating to zero of the first partial derivatives of the total strain energy with respect to the unknown coefficients may not lead to a correct solution of the problem. Pöschl recommends in this case as a constraining condition the equality of the strain energy and the work of the external forces (conservative system).

Two methods may give the solution. The first is to assume the curve or the surface representing in a first approximation the distribution of stresses as a two-dimensional curve function or a three-dimensional surface function of the system of coordinates. This choice may be made in many ways. The simplest may be to base the distribution on experimental tests. The chosen functions possess certain unknown coefficients, which may be determined by the application of the Theorem of Minimum Strain Energy to obtain the proper solution. This approach has its full justification [5] in the proposition put forward by the Author in the previous paper. Before we apply the Minimum Strain Energy Principle we are assured that amongst the chosen set of functions there is one which gives a lower value of the energy than do the others. This method may be considered to be approximate, but it is shorter.

The other approach is to apply the methods of the calculus of variations. It is no longer a problem of seeking an ordinary extremum of a function. This well-known method is thoroughly explained in the literature (Courant, Hilbert, *loc. cit.*, Sokolnikoff, *loc. cit.*, Pöschl, *loc. cit.*). Strictly speaking, the use of the principles of calculus of variations to the solution of problems of this kind was recommended as far back as 1914 (See: Oskar Domke, "Über Variationsprinzipen in der Elastitätslehre nebst Anwendungen auf the technische Statik," *Zeitschrift für Mathematik and Physik*, Mai, 1914, p. 174). Thus the general theoretical way of attacking a problem does exist.[6] But the enormous and insurmountable difficulties in finding exact solutions of differential equations occurring in variational calculus methods applied to such as shells etc. (particularly with cut-outs) may be the reason for an opinion that the question of how to find such a function in any particular case is not yet solved in a rigorous way. Perhaps from an engineering standpoint it may be enough to apply some approximate solutions by use of fast computing machines (punch cards), although the computation difficulties seem to be enormous.

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[5] As an example let us mention that this way was chosen by Prof. Eric Reissner in his very elegant solutions of the shear lag problem. See: E. Reissner, "Least Work Solutions of Shear Lag Problems," *Journal of the Aeronautical Sciences*, Vol. 8, No. 7, May, 1941, p. 284.

[6] Strangely enough, although Courant-Hilbert's book was published in 1931, and Pöschl's paper in 1936, in many books on the theory of elasticity, theory of structures, strength of materials, etc., published later than the above mentioned works, there is nothing said about

The approximate solutions of the problems in the last mentioned case, involving the equating to zero of the first partial derivatives, should be always completed by a thorough discussion of the accuracy of, and estimation of the errors of, the approximation solution.

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the vital and basic importance of distinguishing between these two cases. Also few books mention the necessity of the application of calculus of variations methods. See Sokolnikoff *loc. cit.* There are many examples cited in which the continuous distributed stresses were found by equating to zero the first partial derivatives but in most cases without a thorough discussion of the degree of the approximation.

Houses Built of Earth. MAT KAUTEN. (*Compressed Air Magazine,* Vol. 52, No. 5.)—Here is a design for a home that can be built now when economic conditions have combined to block all manner of construction. The plan meets the present critical housing situation by getting down to earth, the most ancient of building materials. With modern compressed-air tools and streamlined production methods, rammed earth offers a relatively low-cost dwelling.

The technique of tamping earth in forms provides a wall of superior quality without the use of other ingredients. Its characteristics explain its desirability, for such a monolithic structure is proof against fire, dampness, and termites. It requires no insulation whatever because its thermal conductivity is practically zero, and it has a compressive strength of from 18 to 33 tons per cubic foot —strength that increases with age. This is attested to by time, many buildings of this type both here and abroad being hundreds of years old.

Rammed-earth walls need no plastering or painting, though both are possible; and maintenance cost is low enough to be negligible. As for construction, it is a "once-over, all-over" method by which all the separate steps of framing, outside and inside surfacing, and in-between insulation are eliminated. Actually, when the many steps and processing of materials that go on before and after the "show" at a building site are considered, it takes longer to put up a prefabricated, demountable house than to construct one of earth by mechanical means.

Outstanding examples of modern homes of tamped earth are to be found on two government projects that were built within the last ten years by the architect Thomas Hibben. In 1936 he designed seven rammed-earth houses for the Farm Security Administration that were built at Gardendale, Ala., by WPA labor and with materials that represented the smallest percentage of the costs. In 1942 he completed several multiple dwellings near Alexandria, Va. (Federal Works Agency), by the use of air-driven tampers. This was the first attempt towards the mass production of rammed-earth walls and the outcome of progressive experimentation.

The problems in the past have always been the bottlenecks of slow and tedious hand tamping and heavy forms that were back-breaking to lift and readjust after each course. The present-day solutions are compressed-air tamping, easily handled forms, and plans which take the building medium into consideration: unit wall lengths that permit the use of standard forms, arrangement of walls to do away with corners, and windows and doors that extend to the ceiling to eliminate tamping earth over lintels.

R. H. O.

THE MATHEMATICAL DESCRIPTION OF CERTAIN BREAKAGE MECHANISMS LEADING TO THE LOGARITHMICO-NORMAL DISTRIBUTION.

BY

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1. SUMMARY

A number of writers have observed that the particle size distributions obtained from some breakage processes appear to be logarithmico-normal. There have been virtually no attempts to explain this phenomenon. In this paper a statistical model is constructed for a breakage mechanism which will generate size distributions which are asymptotically logarithmico-normal. In this model, $F_n(x)$, the distribution function of the particle sizes, after n steps of the breakage process, is the distribution function of the product of $(n + 1)$ independent random variables and hence, by the central limit theorem, is asymptotically logarithmico-normal.

2. INTRODUCTION.

It has been observed in the analysis of data that arise as a consequence of the breakage of solids that quite often the distribution of sizes arising from the breakage process is logarithmico-normal. The question naturally arises—Can one construct hypothetical breakage mechanisms which have the property of generating a logarithmico-normal distribution? It is our purpose in this paper to describe one such mechanism which has the property of yielding size distributions after breakage which will approximate a logarithmico-normal distribution the longer the breakage process is continued. As far as is known to this writer, this phase of the problem of the breakage of solids has not been treated at all adequately in the literature. The engineering literature on grinding and crushing of solids is extensive (1, 2, 3, 4, 5, 8, 10, 12, 13), but is devoted in the main to attempts to fit size distributions by various empirical curves without attempting to probe more deeply into possible underlying breakage mechanisms. The only mathematical papers on the subject appear to be one by Kolmogoroff (11) and another by Halmos (9). In Kolmogoroff's paper he gives conditions under which a repeated breakage event will lead to a logarithmico-normal distribution. But this paper, in addition to being relatively inaccessible and sketchy, is based on the introduction of a number of complicated functions, and it therefore may be of some interest to present a different approach to the problem which is reasonably simple and direct. There is some connection between the results of Halmos and our own, although our formulation has been framed in such a way that it can be interpreted more directly as a mathematical and physical model of certain types of breakage mechanisms.

3. ASSUMPTIONS CONCERNING THE THEORETICAL BREAKAGE MECHANISM.

Underlying the whole treatment is the concept that any breakage process may be conceived of as composed of discrete steps, and that it makes sense to talk of a "breakage event" which merely consists of a single step in the degradation process. One can then logically study the breakage process after any finite number of steps: $n = 1, 2, \ldots, N, \ldots$. In addition, the following two basic assumptions about the nature of the breakage mechanism will be made: (a) the probability of breakage of any piece during any step of the process is a constant π_1, $0 \leqslant \pi_1 \leqslant 1$, independent of the size[1] of the piece, the presence of other pieces, and the number of steps that have occurred prior to the given step; and (b) the distribution of pieces obtained from the application of a single breakage event to a given piece is independent of the dimension[1] of the piece broken in the sense that the fraction by weight of material having dimension $< ky (0 \leqslant k \leqslant 1)$ arising from the breakage of a unit mass of size y is independent of y.

Condition (b) can be given analytical formulation by saying that if the application of a single step in the breakage process to a unit mass of size y_0 gives rise to a size distribution described by the d.f.[2] $F(x; y_0)$ then a piece of size κy_0 will, after breakage, give rise to a d.f. $F(x; \kappa y_0) = F\left(\frac{x}{\kappa}; y_0\right)$ where $0 \leqslant x \leqslant \kappa y_0$.[3]

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[1] One is not strictly justified in speaking of the size (in the linear sense) or dimension of the piece unless one were dealing with essentially one-dimensional solids such as very thin rods. In actual practice the pieces are three-dimensional and of irregular shape. In this case the logically best procedure would be to classify pieces according to volume since one could at once apply the principle of mass conservation to each piece broken. Unfortunately there are practical difficulties involved in measuring volumes accurately and quickly, and therefore it is common practice to divide a sample into various linear size compartments by sieving through a series of wire mesh screens of decreasing square or circular openings. The pieces in each compartment are then counted, or much more generally weighed (particularly if the pieces are small) and one obtains a size distribution where the fraction of material lying between the linear dimensions $x = x_1$ and $x = x_2$ is given by the weight of material in this size range. In this paper we shall refer to particle size distribution in this sense. This is actually not an important restriction because it is generally accepted that pieces arising from the breakage of solids can be taken as homologous so that the volume of a piece classified as being of linear dimension x has a volume proportional to x^3. It turns out that the main result of this paper, namely, that the size distribution will approach a logarithmico-normal distribution the longer the process continues is independent of the particular size distribution considered (i.e., by volume, linear dimension, count, etc.), since it is a property of the logarithmico-normal distribution that if ξ is a random variable which is logarithmico-normally distributed, then so is ξ^n for any finite n.

[2] I.e., distribution function. For a definition of this concept see (6), page 57. The terminology in this paper is consistent with that in (6).

4. STATEMENT OF THEOREMS.

The first two theorems concerning size distributions are:

Theorem I. Suppose that the initial distribution of pieces is described by the d.f. $G(y)$, $0 \leqslant y \leqslant y_0$ with some finite maximum y_0, and suppose further that the probability of breakage of any piece during any step of the process is independent of the size and is, in fact, equal to one (i.e., $\pi_1 = 1$), and suppose that assumption (b) is satisfied, then the d.f. $F_n(x)$ after n steps in the breakage process is asymptotically logarithmico-normal.

Theorem II. Suppose that the initial distribution of pieces is described by the d.f. $G(y)$, $0 \leqslant y \leqslant y_0$ with some finite maximum y_0, and suppose that assumptions (a) and (b) are satisfied, then the d.f. $F_n(x)$ after n steps in the breakage process is asymptotically logarithmico-normal.

Suppose that condition (a) is replaced by the condition (a′):

(a′). The probability of breakage of any piece during the ith step of the breakage process is a constant π_i[4], independent of the size of the piece (but possibly dependent on i), and of the presence of other pieces, then one can prove Theorem III.

Theorem III. Suppose that the initial distribution of pieces is described by the d.f. $G(y)$, $0 \leqslant y \leqslant y_0$), with some finite maximum y_0, and suppose that conditions (a′) and (b) are satisfied, then the d.f. $F_n(x)$ after n steps of the breakage process is asymptotically logarithmico-normal.

5. PROOF OF THEOREMS.

It is clear that Theorem III contains Theorem II which, in turn, contains Theorem I. We proceed now to the proof of Theorem I. There is no loss in generality if it is assumed from now on that $y_0 = 1$ since this amounts only to a normalization which cannot affect the end result. It is, therefore, assumed that no piece in the initial size distribution exceeds unity and that the initial size distribution, $G(y)$, is a d.f. in the statistical sense taken over the unit interval $0 \leqslant y \leqslant 1$ and $F(x)$ will be the d.f. arising from the breakage of a unit mass of size one.

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[3] It should be mentioned that in proving Theorems I and II which deal with the product of random variables, each having the same distribution, an additional restriction must be imposed on the d.f. $F(x; y_0)$ in order to be able to apply the central limit theorem. This condition is that $\int (\log x)^2 dF(x) < \infty$. (See (6), p. 215.) In theorems I and II it is tacitly assumed that the d.f. $F(x)$ satisfies this condition.

[4] It will be assumed in the sequel that any step for which $\pi_i = 0$ will not be considered a step in the breakage process. Concerning the remaining π_i, it will be assumed that all but a finite number of the π_i exceed some fixed $\delta > 0$. This condition on the π_i plus the finiteness of $\int (\log x)^2 dF(x)$ and $\int (\log x)^3 dF(x)$ is needed in the proof of Theorem III. (See (6), p. 215, in connection with the statement and proof of the Liapounoff theorem.) In Theorem III it is tacitly assumed that the d.f. $F(x)$ satisfies these additional restrictions.

The result of the first step in the breakage process under the assumptions of Theorem I is to give a d.f. $F_1(x)$, $0 \leqslant x \leqslant 1$, which is the d.f. of a random variable ξ_1, $0 \leqslant x \leqslant 1$, which is the product of the independent random variables η_1 and ζ_1 having d.f.'s $G(y_1)$, $0 \leqslant y_1 \leqslant 1$, and $F(z_1)$, $0 \leqslant z_1 \leqslant 1$, respectively. That $\xi_1 = \eta_1\zeta_1$ is a direct consequence of the assumption (b), which assures the independence of η_1 and ζ_1. Similarly $F_2(x)$, the result of the first two steps of the breakage process, is the d.f. of a random variable ξ_2, which is the product of two independent random variables η_2 and ζ_2 having d.f.'s $F_1(y_2)$, $0 \leqslant y_2 \leqslant 1$, and $F(z_2)$, $0 \leqslant z_2 \leqslant 1$, respectively. But $F_1(y_2)$, $0 \leqslant y_2 \leqslant 1$, is the d.f. of a random variable η_2, which is itself the product of two independent random variables η_1 and ζ_1. Therefore, $F_2(x)$ is the d.f. of a random variable $\xi_2(0 \leqslant x \leqslant 1)$ which is a product of three independent random variables η_1, ζ_1, and ζ_2, where η_1 has d.f. $G(y_1)$, $0 \leqslant y_1 \leqslant 1$, and ζ_1 and ζ_2 have d.f.'s $F(z_1)$, $F(z_2)$, $0 \leqslant z_1 \leqslant 1$, $0 \leqslant z_2 \leqslant 1$, respectively. It is clear that after n steps in the process $F_n(x)$ is the d.f. of a random variable ξ_n, $0 \leqslant x \leqslant 1$, which is a product of $(n + 1)$ independent random variables $\eta_1, \zeta_1, \zeta_2, \ldots, \zeta_n$, where η_1 has d.f. $G(y_1)$, $0 \leqslant y_1 \leqslant 1$ and the ζ_i each have d.f.'s $F(z_i)$, $0 \leqslant z_i \leqslant 1$, for $1 \leqslant i \leqslant n$. Therefore:

$$\log \xi_n = \log \eta_1 + \sum_{i=1}^{n} \log \zeta_1 \to \log \zeta_i. \tag{1}$$

By the central limit theorem of statistics (6, p. 213) it follows that $\log \xi_n$ is a random variable which is asymptotically normal. Therefore, ξ_n is a random variable which is asymptotically logarithmico-normal. This completes the proof of Theorem I.

It is useful in applications to have an explicit formula for $F_n(x)$, the d.f. after n steps of the breakage process. This formula can be given as a recursion formula:

$$F_n(x) = \int_0^1 F\left(\frac{x}{y}\right) dF_{n-1}(y), \tag{2}$$

where

$$F_1(x) = \int_0^1 F\left(\frac{x}{y}\right) dG(y).^{5} \tag{3}$$

It can be readily shown that the mean and variance of $F_n(x)$ are given

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[5] The formulae (2) and (3) give another analytical statement of the fact that $F_n(x)$ and $F_1(x)$ are products of random variables. It is known that if ξ and η are two positive random variables whose d.f.'s are respectively $F(x)$ and $H(y)$, then the product $z = \xi\eta$ is a random variable whose d.f. $H(z)$ is given by:

$$H(z) = \int_0^\infty F\left(\frac{z}{y}\right) dG(y).$$

See e.g., (7).

by $m_1m_2^n$ and $(\sigma_1^2 + m_1^2)(\sigma_2^2 + m_2^2)^n - m_1^2m_2^{2n}$, respectively, where m_1 and σ_1^2 are the mean and variance of $G(y)$ and m_2 and σ_2^2 are the mean and variance of $F(x)$.

To prove Theorem II we proceed as follows: suppose that the probability of breakage of any particle in a single step of the breakage process is π_1, $0 \leqslant \pi_1 \leqslant 1$, independent of the size of the particle. Then $\kappa_1 = 1 - \pi_1$ is the probability of not breaking. Again it is assumed that the initial size distribution is $G(y)$, $0 \leqslant y \leqslant 1$, and that $F(x)$ is the d.f. resulting from the breakage of a unit mass of size one. We now make the observation that a process for which (a) the probability of breakage of each particle is certainty and (b) the d.f. arising from the breakage of a unit mass of size y is given by:

$$H\left(\frac{x}{y}\right) = \pi_1 F\left(\frac{x}{y}\right) + \kappa_1 A\left(\frac{x}{y}\right), \tag{4}$$

where

$$\begin{aligned} A(x) &= 0,\ 0 \leqslant x < 1 \\ &= 1,\ x \geqslant 1 \end{aligned}$$

is equivalent to a process satisfying the conditions of Theorem II. The reasoning used in proving Theorem I can then be applied and it will follow that $F_n(x)$, the d.f. after n steps in the breakage process, is the d.f. of a random variable ξ_n, $0 \leqslant x \leqslant 1$, which is the product of $(n + 1)$ independent random variables $\eta, \tau_1, \tau_2, \ldots, \tau_n$ where η has d.f. $G(y)$, $0 \leqslant y \leqslant 1$, and the τ_i each have d.f.'s $H(t_i)$, $0 \leqslant t_i \leqslant 1$ for $1 \leqslant i \leqslant n$. As in the proof of Theorem I, we can now apply the central limit theorems of statistics to prove that ξ_n is a random variable which is asymptotically logarithmico-normal and this completes the proof of Theorem II.

An explicit formula for $F_n(x)$ is given by:

$$\begin{aligned} F_n(x) = \pi_1^n F_n^*(x) + C_1^n \pi_1^{n-1}\kappa_1 F_{n-1}^*(x) + C_2^n \pi_1^{n-2}\kappa_1^2 F_{n-2}^*(x) \\ + \cdots + C_m^n \pi_1^{n-m}\kappa_1^m F_{n-m}^*(x) + \cdots + \kappa_1^n G(x), \end{aligned} \tag{5}$$

where $F_n^*(x)$ is defined by the equation:

$$F_n^*(x) = \int_0^1 F\left(\frac{x}{y}\right) dF_{n-1}^*(y) \tag{6}$$

and

$$F_1^*(x) = \int_0^1 F\left(\frac{x}{y}\right) dG(y). \tag{7}$$

In the case of Theorem III we are dealing with a process for which the probability of breakage during the ith step of the process is a constant π_i, independent of the size of the piece, but possibly dependent on i. Again we start with an initial size distribution given by the d.f. $G(y)$, $0 \leqslant y \leqslant 1$, and assume that $F(x)$ is the d.f. resulting from the breakage of a unit mass of size one. It is readily seen that such a breakage process is equivalent to a breakage process for which (a) the probability of breakage of each particle is certainty and (b) the d.f. arising from the breakage of a unit mass of size y in the ith step of the breakage process is given by:

$$H_i\left(\frac{x}{y}\right) = \pi_i F\left(\frac{x}{y}\right) + \kappa_i A\left(\frac{x}{y}\right), \tag{8}$$

where

$$\begin{aligned} A(x) &= 0,\ 0 \leqslant x \leqslant 1 \\ &= 1,\ x \geqslant 1 \end{aligned}$$

and

$$\kappa_i = 1 - \pi_i.$$

The reasoning used in proving Theorem I can be applied and, therefore, $F_n(x)$, the d.f. after n steps in the breakage process, is the d.f. of a random variable ξ_n, $0 \leqslant x \leqslant 1$, which is the product of $(n + 1)$ independent random variables η, τ_1, τ_2, . . ., τ_n where η has d.f. $G(y)$, $0 \leqslant y \leqslant 1$, and the τ_i have d.f.'s $H_i(t_i)$, $0 \leqslant t_i \leqslant 1$, $1 \leqslant i \leqslant n$. We can now apply the central limit theorem to prove that ξ_n is a random variable which is asymptotically logarithmico-normal and this completes proof of Theorem III.

The d.f. $F_n(x)$ after n steps is related to the d.f. $F_{n-1}(x)$ after $(n - 1)$ steps by the equation:

$$F_n(x) = \pi_n \int_0^1 F\left(\frac{x}{y}\right) dF_{n-1}(y) + \kappa_n F_{n-1}(x), \tag{9}$$

where

$$F_1(x) = \pi_1 \int_0^1 F\left(\frac{x}{y}\right) dG(y) + \kappa_1 G(x). \tag{10}$$

6. PLAUSIBILITY OF BREAKAGE MECHANISM PROPOSED IN THIS PAPER.

Theorems I, II and III explain in a compact way why so many size distributions arising from repeated crushing and fine grinding appear to be logarithmico-normal (1, 10, 13). For such processes condition (a) may be a reasonable approximation to the true state of affairs (in fact, the probability of breakage of each particle in a single breakage step may in the case of crushing well be equal to one, i.e., certainty). Furthermore, assumption (b) seems quite plausible and is, in fact, tacitly assumed to be true by a number of authors. (See e.g., (8).) It should be mentioned that other writers (2, 3, 4, 5, 8) have made the observation that a degradation process may be considered as a sequence of breakage events. But, all of these writers have missed the essential point that under certain hypotheses the laws of probability operate in such a way that size distributions obtained from the continued repetition of the breakage process are asymptotically logarithmico-normal.

7. REMARK ON SITUATION WHEN CONDITIONS OF THIS PAPER ARE NOT SATISFIED.

The question of what happens in a repeated breakage process if either condition (a) (or generally (a′)) or (b) is not satisfied is an interesting one. A number of fragmentary results concerned with the effect of changes in assumption (b) have been obtained. The details are left for a subsequent publication.

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Tube Both Heats and Cools Air. (*Compressed Air Magazine*, Vol. 52, No. 5.)—A hot-cold pipe that converts ordinary compressed air into both hot and cold air without the aid of moving parts has been built by Westinghouse research engineers for study purposes. As yet they have no plausible explanation of the phenomenon, although it is conjectured that it may be attributable to the frictional effect of air currents moving at different velocities.

Compressed air is admitted at one end of the pipe through a nozzle that causes it to strike the inner wall tangential to its diameter, thereby creating a whirlpool effect. The air in the center becomes cold almost instantly, while that along the periphery gets warm. The cold air is drawn off at one end through a small opening in the center, and the warm air from the outer space.

The tube was invented by R. Hilsch of Germany. Using a pipe 3/10 inch in diameter, he claims to have obtained simultaneously jets of air at 154° and 10° F. The Westinghouse model, was purposely constructed larger to facilitate instrumentation for study purposes. It is 1 inch in diameter and 15 inches long, and gives a temperature difference of only a few degrees.

The Westinghouse announcement states that "low efficiency rules out commercial application at this time, but like the 'blue glow' in Edison's experimental electric light that led to his invention of the first electronic tube, it may lead to unexpected developments. Even now it holds promise as a handy laboratory tool for making cold as simply as a Bunsen burner produces heat."

R. H. O.

Gas Turbine Electric Locomotive. (*Electrical Engineering*, Vol. 66, No. 5.)—A gas turbine electric locomotive is to be designed and built at Baden, Switzerland, for the Great Western Railway. The order for the locomotive has been placed with the British Brown-Boveri, Ltd.

The locomotive will be rated at 2,500 horsepower with a maximum continuous speed of 90 miles per hour, capable of hauling a trailing weight of 1,200 tons at speeds up to 40 miles per hour continuously. Starting tractive effort is to be 33,000 pounds. Length of the locomotive will be 65 feet, 6 inches, and the weight will be 113 tons, including enough fuel oil to make a trip of 250 miles.

Air at 45 pounds per square inch gauge will be delivered by a turbine-driven compressor through an exhaust gas heat exchanger to a combustion chamber for oxidation of the fuel. The greater portion of the compressed air will be bypassed to be mixed with the hot products of combustion, cooling them to approximately 1,100° F. before the gas is expanded through the turbine. Exhaust gases will pass through the heat exchanger where sufficient heat will be recovered to raise the temperature of the air delivered to the combustion chamber to 500° F.

Total power produced by the turbine unit will be 10,300 horsepower, 7,800 of which will be absorbed by the air compressor. A 2,500 horsepower d-c generator will be driven through a speed reduction gear to supply series traction motors mounted on the outer axles of each of the two 6-wheel trucks. Control is to be accomplished from either of two cabs at opposite ends of the locomotive.

A less powerful locomotive of the same type was built five years ago by Brown-Boveri for the Swiss Federal Railways.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

ELECTROLESS PLATING ON METALS BY CHEMICAL REDUCTION.

A new method for plating nickel and cobalt on metal surfaces without the use of electric current has been developed by Abner Brenner and Grace E. Riddell of the National Bureau of Standards. This process, known as *electroless plating*, is brought about by chemical reduction of a nickel or cobalt salt with hypophosphite in hot solution. The reaction is catalytic, and under the prescribed conditions of concentration and pH, no plating occurs unless certain metals, such as steel or nickel, are introduced in the bath. The reduction then occurs only on the surface of the immersed metal with the production of an adherent coating of 93 to 97 per cent. purity.

Photomicrographs of deposits obtained in this way show both a laminar and a columnar structure, similar to bright nickel electrodeposits. The electroless deposits are of good quality—sound though brittle, and usually bright. Since they can be made as hard as tool steel, the method may prove useful where hard, wear-resistant surfaces are required, as in bearings. The process is particularly applicable to the plating of recesses, irregular-shaped objects, and enclosed areas such as tubes, where a centered internal electrode with special current leads would be needed in electroplating.

The equipment is simple and more easily assembled than that required for electroplating. No generators, rheostats, special racks, or contacts are necessary. Small parts which cannot be barrel-plated economically are readily plated by the electroless process if suspended by a string or in a bag affording ample exposure of the metal surface to the solution. There is no need of constant motion, as in barrel plating, since current distribution is not involved.

While electroless deposits of cobalt and cobalt-nickel alloys have been obtained only from ammoniacal solutions, nickel can be deposited from either acid or alkaline solutions. The reactions, requiring a temperature above 90° C., are given in the following equations (in which cobalt may be written for nickel):

$$NiCl_2 + NaH_2PO_2 + H_2O \rightarrow Ni + 2HCl + NaH_2PO_3$$

or

$$NaH_2PO_2 + H_2O \rightarrow NaH_2PO_3 + H_2.$$

The first reaction is the important one, resulting in the deposition of nickel; the second reaction tends to lower the efficiency of the process through oxidation of the hypophosphite.

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* Communicated by the Director.

A unique feature of the electroless process is the catalytic initiation of the reaction by the following metals: Iron or steel, nickel, gold, cobalt, palladium, and aluminum. Unless one of these metals is introduced into the solution, no reaction takes place. Once started, the reaction continues at the metallic surface and only rarely occurs in other parts of the bath. For this reason, the containing vessel should be of glass, plastic, or other noncatalytic material.

Objects to be plated are cleaned by any of the accepted procedures and are given an acid dip before being suspended in the hot solution. The rate of deposition is about the same as in barrel electroplating, ranging from 0.0002 to 0.0008 inch per hour, depending upon the type of solution used. During the process, the pH must be kept within a certain range, and if the operation is of long duration, the nickel salt and hypophosphite are replenished at regular intervals.

The composition of the plating bath may vary within rather wide limits. Suggested solutions are given in table 1. In addition to the metal ions and hypophosphite, the alkaline solutions contain (1) hydroxycarboxylic salts and ammonium salts to prevent precipitation of the metal salts, and (2) certain alkalies to regulate the pH. In the acid nickel solutions, organic acids are used as buffers.

TABLE 1.

Suggested Compositions for the Various Electroless Solutions.

	Nickel		Cobalt	Cobalt-Nickel
	Alkaline	Acid		
	g/l	*g/l*	*g/l*	*g/l*
Nickel chloride, $NiCl_2 \cdot 6H_2O$	30	30		30
Cobalt chloride, $CoCl_2 \cdot 6H_2O$			30	30
Sodium hypophosphite, $NaH_2PO_2 \cdot H_2O$	10	10	20	20
Sodium citrate, $Na_3C_6H_5O_7 \cdot 5\frac{1}{2}H_2O$	100		35	
Rochelle salt, $NaKC_4H_4O_6 \cdot 4H_2O$				200
Sodium hydroxyacetate, $NaC_2H_3O_3$		50		
Ammonium chloride, NH_4Cl	50		50	50
Alkali for neutralizing	NH_4OH	NaOH	NH_4OH	NH_4OH
pH	8-10	4–6	9–10	8–10
Rate of deposition, mm/hour	0.008	0.015	0.015	0.015
Rate of deposition, inch/hour	0.0003	0.0006	0.0006	0.0006
Appearance of deposit	Bright	Semi-bright	Dull	Semi-bright

Electroless plating on noncatalytic metal surfaces may be accomplished in two ways. If a film of palladium or rhodium of nearly monatomic thickness is first applied by chemical replacement on a noncatalytic metal, deposition of nickel or cobalt will occur on the activated surface. Electroless plating of copper may be carried out in this way. A second method of initiating the reduction is to bring a less noble metal, such as iron or aluminum, in contact with the noncatalytic metal while it is immersed in the hot electroless solution. Once the process has been started, it continues because of the catalytic action of the initial deposit.

As formed, the electroless nickel deposits are brittle but become ductile when heated. These deposits are harder than the ordinary electrodeposited nickel, and upon heating their hardness is still further increased. This is in contrast to the behavior of "hard" electrodeposited nickel, which has high initial hardness but softens upon heating. The hardening may be explained as "precipitation hardening," probably of phosphides.

The adhesion of the nickel deposit to mild steel is such that it cannot be flaked off by bending, but on high-carbon steel this property is less satisfactory. In salt-spray tests on steel coated with 0.0002, 0.0005, and 0.001 inch of electroless nickel, in comparison with similar panels coated with electrodeposited nickel, the protective value of the two types of coatings was virtually the same. However, the electroless cobalt does not compare so favorably with electrodeposited cobalt.

The yield or efficiency of the reduction based on the decomposition of hypophosphite is 37 and 66 per cent. for nickel and cobalt, respectively. About 2 grams of nickel or nearly 4 grams of cobalt are reduced by 10 grams of sodium hypophosphite. Because of the moderate yield and the present high cost of sodium hypophosphite, the process is expensive. Extensive commercial use of electroless plating is thus dependent upon a reduction in the price of this chemical.

NOTE: Recent developments in electroless plating were presented in a paper at the convention of the American Electroplaters' Society in Detroit on June 23, 1947. Earlier phases of this work have been reported in Bureau Research Paper RP 1725, "Nickel Plating on Steel by Chemical Reduction," Abner Brenner and Grace E. Riddell, *J. Research NBS*, **37**: 1 (1946).

NEW STATISTICAL METHOD OF PREDICTING SUNSPOTS AIDS RADIO PROPAGATION FORECASTS.

The prediction of solar activity, which greatly affects radio communication and is evidenced by spots on the sun, has been advanced through the application of a new statistical method, by A. G. McNish and Virginia Lincoln of the National Bureau of Standards. The new technique, depending on available sunspot data for a number of previous 11-year cycles, has a sounder scientific basis than former methods of prediction. Moreover, it is expected to be applicable to a wide variety of cyclical phenomena, such as long-term weather variations and climatic changes.

Accurate sunspot prediction became necessary at the Bureau in connection with its radio prediction service, which each month issues radio propagation predictions three months in advance. This information enables radio engineers to calculate the best usable frequencies for communication between any two points in the world for any hour of the day.

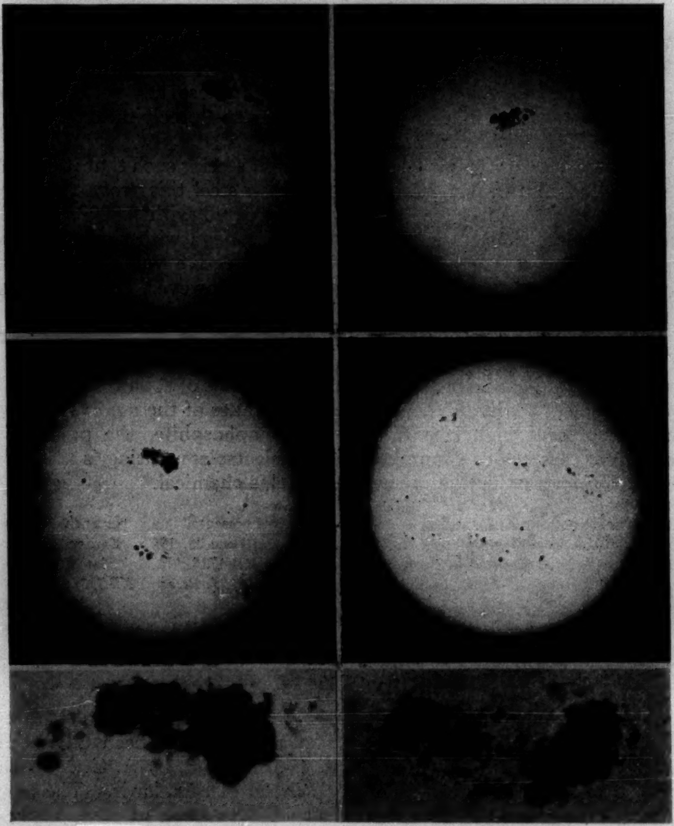

FIG. 1. Photographs of sunspots of the present cycle (from plates furnished by the U. S. Naval Observatory). Upper left, February 1946; upper right, July 1946; middle left, March 1947. (These three pictures include the largest sunspot groups ever photographed.) Middle right, May 1947, representing the highest monthly sunspot number in over 100 years, according to scientists at the National Bureau of Standards. Lower left, enlarged view of sunspot group of March 1947. Lower right, large group of April 1947, considered by some authorities to be a later phase of the March group.

Long-distance radio transmission is made possible by the ionosphere, a series of layers in the atmosphere 50 to 250 miles above the earth. Radio waves sent out near the earth's surface travel in straight lines until they reach the ionosphere and are reflected back to the earth, just as light striking a mirror is reflected. This reflection of radio waves is due to the fact that the ionosphere is made electrically conducting as a result of the ionization of gas molecules in its layers by ultraviolet light from the sun.

The ionosphere continually exhibits fluctuating characteristics due to changes in the amount of ultraviolet light from the sun, which varies greatly over an 11-year period. The number and activity of sunspots—disturbances in the outer layers of the sun indicating local variation of the sun's temperature—serve as a measure of the relative intensity of radiation from the sun. When there is an increase in sunspots, the average electrification of the ionosphere increases, so that high frequencies that otherwise would pass through into space are reflected back to the earth. Thus, in order to predict radio propagation conditions, the long-time variation in solar activity must also be forecast.

With the discovery of the close relationships existing between radio propagation and sunspot activity, the prediction of sunspot numbers assumed great practical importance. Methods of prediction have covered a broad range, including harmonic analysis, assumption that cycles repeat after a number of years, and numerous empirical relationships between heights of maxima, rate of rise, and similar properties of the data. However, the various formulas developed by these methods have led to widely different predictions.

In the development of a satisfactory sunspot prediction formula at the National Bureau of Standards, it was assumed that (1) in a time series exhibiting cyclical tendencies, a first approximation to a future value is the mean of all past values for the same stage of the cycle, and (2) this approximation can be improved by adding to the mean a correction proportional to the departure of earlier values of the same cycle from their respective means. The second assumption is justified by the observed tendency in sunspot numbers for annual deviations from the mean to have the same sign and similar magnitudes in consecutive years.

The prediction formula then becomes

$$R_n' = \bar{R}_n + k_{n-1}\Delta R_{n-1} + k_{n-2}\Delta R_{n-2}\cdots,$$

where R_n' is the predicted value in a particular cycle, $\bar{R}_n$ the mean of all corresponding values in preceding cycles, ΔR_{n-1} the deviation of the particular R_{n-1} for this cycle from the mean of all R_{n-1}'s from previous cycles, and the k's are proportionality constants.

The least-squares criterion that the sum of the squared deviations from the mean be a minimum was used in evaluating the proportionality

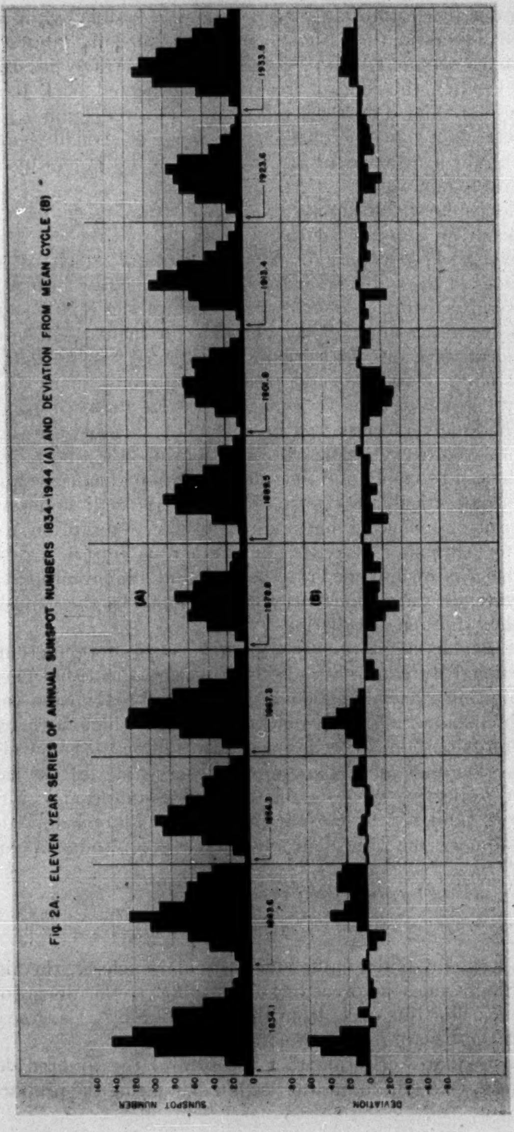

Fig. 2A. ELEVEN YEAR SERIES OF ANNUAL SUNSPOT NUMBERS 1834-1944 (A) AND DEVIATION FROM MEAN CYCLE (B)

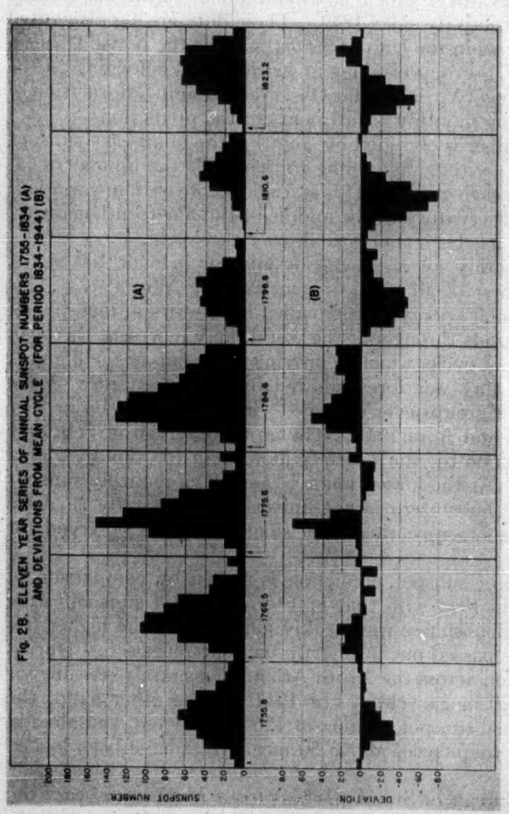

FIG. 2. Eleven-year series of annual sunspot numbers and deviations from the mean cycle for the period 1834 to 1944. Figure 2A is a somewhat idealized representation of the data from 1834 to 1944 obtained by spacing the minima of successive cycles at 11-year intervals and plotting the observed annual values at these minima and for the 10 succeeding years. The conservatism of the series, upon which the new prediction system developed at the National Bureau of Standards is based, is evident from the tendency of the adjacent annual deviations to have the same sign and similar magnitudes. It is also clear that this tendency decreases as the separation in time of the deviations in any cycle is increased, so that the further in advance a prediction is made, the greater its uncertainty. Figure 2B is a similar representation of the data from 1755 to 1834, the deviations being taken from the mean cycle for the period 1834 to 1944. These deviations from the mean cycle are noticeably greater than those subsequent to 1834, probably because of the absence of a uniform system of observations and reductions from one observer to another during the earlier period.

constants. Upon comparison of observed values with those predicted by this method, it was apparent that the best prediction is usually obtained by setting all of the k's except the one for the previous year equal to zero, and this procedure is followed in most cases.

The basic information on sunspot numbers is obtained from the Zürich (Switzerland) Observatory, which has maintained continuous records on all types of solar activity since 1849. These data are supplemented by professional sources in the United States, such as the Naval Observatory. In addition, a group of amateur astronomers belonging to the American Association of Variable Star Observers submits information of great value in making monthly adjustments to the predictions.

The sunspot number is obtained by counting the number of sunspot groups, multiplying by ten, and adding to the result the number of individual sunspots in each group. This statistical convention was adopted at the Zürich Observatory in the middle of the nineteenth century, and since that time has been standard all over the world.

The highest monthly average sunspot number in over 100 years, and one of the highest of all time, occurred during May 1947. This abnormal burst of solar energy, accompanied by a series of ionospheric disturbances of all kinds throughout the world, may represent the peak of the current sunspot cycle, which is proving to be one of the highest. The high value in May was foreshadowed by the appearance of the three largest sunspot groups ever observed—in February 1946, in July 1946, and in March and April 1947. The group observed in February 1946 covered an area on the sun's surface more than 100 times the area of the earth's disc. At the lowest point of a cycle, a monthly sunspot number of zero or almost zero is not unusual. As recently as April 1944, only one sunspot, appearing for a single day, was observed for the entire month.

The importance of sunspot prediction is shown by comparison of radio transmission in 1944 with that in 1947. In 1944 sunspots were at a minimum, the ionosphere was weakly ionized, and the higher frequency radio waves passed out into space without reflection. During the year, transmission across the North Atlantic was rarely possible for frequencies above 20 megacycles. For 1947, on the other hand, the extremely high annual sunspot number of 126 is predicted, and already transmissions using frequencies above 50 megacycles have been logged over this path.

The National Bureau of Standards was for a number of years the only government agency engaged in systematic ionospheric research. As early as 1910, scientists at the Bureau had investigated the correlation of ionospheric changes with radio transmission and reception. After an aircraft disaster in the European theater in 1941 had shown the importance of radio propagation information in the success of military operations and the saving of lives, ionospheric investigations were intensified, and the Interservice Radio Propagation Laboratory was set up at the Bureau to provide the required information for the Army and Navy. At the close of the war, this activity was transferred to the Bureau completely and given divisional status as the Central Radio Propagation Laboratory. By 1946, radio propagation predictions and other research results were made available to the interested public as well as the Armed Forces. These services are found to be as valuable to commercial radio interests as they have been to the Army and Navy.

Daily "soundings" of the ionosphere are taken all over the world by an international network of 58 ionosphere stations, 14 of which are operated or supported by the Bureau. These daily soundings measure the critical frequency (the limiting frequency for reflections back to the earth), absorption of radio energy (an indication of the power required to transmit a given frequency over a particular distance), and the heights of the various layers (determined through the use of radar-like echo equipment). The sunspot predictions are correlated with this information to provide the working data used at the Bureau in predicting radio propagation characteristics.

Groups now using the service include airline companies, steamship lines and the merchant marine, television and radio schools, American and foreign universities, radio and telegraph companies, manufacturers of communication equipment, consulting radio engineers, press wireless and telegraph services, radio magazines, broadcasting companies both here and overseas, industrial electrical firms, navigation instrument companies, research laboratories, electric power companies, and geophysical exploration organizations.

In the Federal Government, the Army and Navy are, of course, the principal users, the service having been established originally for their needs. Other important users in the Government are the Weather Bureau, the State Department, the Federal Communications Commission, the Civil Aeronautics Administration, the Forest Service Radio Laboratory, the Coast Guard, and the Coast and Geodetic Survey.

Dwindling Resources Cost of War in Materials. (*Electrical Engineering*, Vol. 66, No. 5.)—One of the war's great costs to the United States is turning out to be in the basic materials that support the nation's strength according to a series of official reports bringing to light the real extent of cost in resources. Julius A. Krug, Secretary of Interior, is suggesting a $1,000,000 survey to determine exactly how much mineral wealth is left. The United States Forestry Service is recommending tighter rules on the use of timber resources.

Metal resources are depleted to a great extent because the best grade ores are gone and over five billion tons of United States minerals went into the war. Oil wells in the United States were tapped for about eight billion barrels of oil in the war period, and in the closing months of the war could not keep step with demand. Timber stocks in some of the essential categories are almost gone after three centuries of plenty. Soil resources suffered from being overcropped and certain areas are approaching status of wasteland.

However, substitute resources were improved by the pressure of war demand. A synthetic rubber industry was built to turn out one million tons of rubber a year. And by using sea water magnesium output was increased 75 times to a 375 million pounds per year output.

Iron ore is the most important of the mineral resources and the United States has an abundant supply for future needs, but the supply of high grade ores may last less than 17 years. Today the United States gets about half of its copper from abroad, mainly because its own deposits are progressively harder to mine. Lead and zinc deposits are reported to have only a few years to last at present rates of expenditure and the result already has been felt in a scarcity of storage batteries, paint, and good gasoline for automobiles.

Higher costs for raw materials are ahead. More goods are needed to provide for a larger population. More supplies must come from farther away because United States resources can be expanded now only by digging deeper, utilizing low-grade ores, and expanding synthetics. All these methods are being used and all are more expensive.

R. H. O.

Paradox of Plenty.—"If we choose to build and conserve our resources—as we must in order to survive—then we have no alternative but to accept abundance and learn to live with it," Secretary Anderson told the House Committee considering long-range agricultural policy.

"It must sound paradoxical to countries poor in resources that we must search for ways to use our abundance in order to provide abundance for the future. And yet that is our true situation. Our experience has shown us that the shortest road to national destruction—the quickest way to become a have-not nation—is the route we follow when consumption goes down, surpluses pile up, and producers must neglect and drain the natural resources in order to live. It is not through using our agricultural resources that we impoverish ourselves but through misusing them, I doubt that any other nation has ever had such an opportunity as we have today—to use resources without misusing them, and by producing an adequate supply of what the people want to assure plentiful resources for the future."

R. H. O.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, WEDNESDAY, NOVEMBER 19, 1947.

The Stated Monthly Meeting of The Franklin Institute was held on Wednesday, November 19th, at 8:15 p.m. The meeting was called to order by Mr. Richard T. Nalle, President of the Institute. There were approximately 200 persons in attendance.

The Secretary then gave his report on the membership for the month of October. The following members were admitted:

Active	45
Associate	28
Student	19

The Total Institute Membership as of the month of October was 5411.

The President then introduced the speaker of the evening, Dr. Ellice McDonald, Director of the Biochemical Research Foundation of The Franklin Institute who spoke on the "Effect of Radiation on Cells." Dr. McDonald discussed the results of experiments which have been carried on at the Biochemical Research Foundation for the past fifteen years on the effect of ultra-violet, X-rays and particularly neutrons on the mortality, cancer production, blood tissue of animals. The use of a radioactive material as a tracer was demonstrated.

At the conclusion of Dr. McDonald's talk, the President dismissed the audience with a rising vote of thanks to the speaker of the evening.

HENRY B. ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, November 12, 1947.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, NOVEMBER 12, 1947.

MR. WALTER C. WAGNER in the Chair.

The following reports were presented for final action:

No. 3130: Work of Edwin H. Colpitts.

This report recommended the award of an Elliott Cresson Medal to Edwin Henry Colpitts, of Orange, New Jersey, "In consideration of his pioneering achievements leading to the development of practical systems of long distance communication, both by wire and radio."

No. 3167: "Teledeltos."

This report recommended the award of an Edward Longstreth Medal to Raleigh J. Wise, of Dunellen, New Jersey, "In consideration of the development of a dry electrosensitive recording blank which can be stored and handled like ordinary paper with practically no deterioration or change and which can be permanently marked by simple means."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Abstract of **Effect of Borate on a Protein-Polysaccharide Complex, the Phosphoesterase from Calf Intestinal Mucosa.**—CHARLES A. ZITTLE (*Journal of Biological Chemistry*, **167**: 297, 1947). In an attempt to fractionate a preparation of phosphodiesterase isolated from calf intestinal mucosa, borate was added prior to ammonium sulfate precipitation. The solubility of the preparation was reduced by the addition of the borate and a polysaccharide-rich, enzyme-poor fraction could be precipitated by one-half saturation with ammonium sulfate. The enzyme fraction, which could be precipitated at a higher concentration of ammonium sulfate, was twice as active, on a unit weight basis, as the starting material. Phosphodiesterase activity and phosphomonoesterase (alkaline phosphatase) activity were reversibly inhibited approximately 50 per cent. by 0.01 M borate. The data suggest that the enzyme may be a protein-polysaccharide complex.

Abstract of **Enzymatic Hydrolysis of Desoxyribonucleic Acid.**—CHARLES A. ZITTLE (*Archives of Biochemistry*, **13**: 191, 1947) Four secondary phosphoric acid groups were found to be liberated per tetranucleotide-residue in the hydrolysis of desoxyribonucleic acid by the successive action of specific nuclease and phosphoesterase, measured by manometry and titrimetry. Phosphoesterase alone hydrolyzed desoxyribonucleic acid slowly to the extent of 70 per cent., but after preliminary treatment of the nucleic acid with the specific nuclease the action of the phosphoesterase was rapid and hydrolysis was complete.

Abstract of **Effect of Sodium Arsenate on Phosphoesterase from Calf Intestinal Mucosa.**—CHARLES A. ZITTLE, LORNA A. WELLS AND WM. G. BATT (*Archives of Biochemistry*, **13**: 395, 1947). The course of hydrolysis of ribonucleic and desoxyribonucleic acids by phosphoesterase from calf intestinal mucosa in the presence of sodium arsenate (0.002 M) has been determined. The break-down of both the diesters of phosphoric acid, polynucleotides, and the monoesters, mononucleotides, was inhibited, but the inhibition of the latter greatly exceeded the former. With desoxyribonucleic acid, hydrolysis of the mononucleotides is almost completely prevented, allowing their formation to proceed completely. Manometric studies of the inhibition of the first stage of hydrolysis indicate that this phenomenon is not due to competition with the substrate. The procedure utilized for determining phosphoric acid in the presence of arsenate is described.

Abstract of **Hydrolysis of Ribonucleic and Desoxyribonucleic Acids with Phosphoesterase from Calf Intestinal Mucosa.**—CHARLES A. ZITTLE (*Federation Proceedings*, **6**: 500, 1947). A phosphoesterase-preparation from calf intestinal mucosa hydrolyzes both diesters and monoesters of phosphoric acid, hydrolysis of the latter being a measure of alkaline phosphatase. It is possible that a single enzyme carries out both reactions. Methods are described for determining the diesterase, using ribonucleic acid as the substrate and for differentiating between this non-specific phosphodiesterase and ribonucleinase.

The secondary phosphoric acid groups released by phosphoesterase from ribonucleic acid have been measured manometrically and titrimetrically. The same methods have been used to measure the acid groups released from desoxyribonucleic acid by a specific nuclease and subsequently by the phosphoesterase, although the phosphoesterase was found to partially hydrolyze desoxyribonucleic acid without the intervention of the nuclease.

The effect of arsenate on the hydrolysis of both nucleic acids to mononucleotides and subsequently to nucleosides is described. In the case of desoxyribonucleic acid complete inhibition of the latter stage by arsenate provides a method for obtaining mononucleotides of this nucleic acid.

The diesterase is inhibited by borate and phosphate. With phosphate inhibition appears to be by competition with the substrate but with borate some other mechanism must be involved, perhaps interaction with a polysaccharide component of the enzyme.

Interaction of borate with one polysaccharide-rich component of the phosphoesterase preparation has been demonstrated electrophoretically and by an increase in viscosity. This interaction of borate and polysaccharide, which has been useful for purifying the enzyme, may be also of more general importance.

BOOK REVIEWS.

SCIENCE SINCE 1500, A Short History of Mathematics, Physics, Chemistry, Biology, by H. T. Pledge. 357 pages, illustrations, 15 × 24 cms. New York, Philosophical Library, 1947. Price $5.00.

Written by the librarian of the Science Museum of London, one purpose of this publication is to serve as a background study for the various manuals on the history of special scientific subjects, published by the Museum. It aims also to serve as a coordinating survey of the history of modern science, incorporating the results of various studies on hitherto obscure points.

The treatment of the material is generally by centuries with broad subject groupings, but the nineteenth century has been subdivided with more specific headings. To bring the material within the compass of an ordinary volume much compression has been necessary. Consequently no effort has been made to detail the vast masses of facts available at any one period, although exposition has been given of the main principles. The human sciences, sociology, anthropology, psychology and economics are not considered and biographical data has been kept at a minimum. There remains however a vast amount of information which has been succinctly presented to give a comprehensive view of the development of modern science.

A short bibliographical note and detailed subject and name indexes add greatly to the usefulness of the volume. American readers are indebted to the publisher for making this English publication readily available to them.

G. E. PETTENGILL.

THE NAMING OF THE TELESCOPE, by Edward Rosen. 110 pages, illustrations, 12 × 18 cms. New York, Henry Schuman, 1947. Price $2.50.

In this slender volume Mr. Rosen has investigated the problem of the naming of the telescope. Over the years two rival traditions have grown up, with successive writers following one or the other of the two traditions often, apparently, with no attempt to evaluate the evidence. These two protagonists are Frederick Cesi and John Demisiani, neither of whom seems to have made any pretension to the honor, but who rather have had it claimed for them.

Mr. Rosen has reviewed all the evidence and arrives at the conclusion that the sources attributing the naming to Demisiani are more reliable. His presentation of the evidence and his astuteness in interpreting clues gives one the feeling of participating in a piece of detective work. The study is fully documented for those who care to refer to the original sources.

In conclusion one cannot do better than to quote from Dr. Harlow Shapley's foreword: "this is a most attractive treatise by an expert who has trod deftly through the tanglewood of books and correspondence and who has, from the fragments of the past made a seemingly small subject noble and exemplary."

G. E. PETTENGILL.

MATHEMATICS AS A CULTURE CLUE AND OTHER ESSAYS, by Cassius Jackson Keyser. 277 pages, illustrations, 14 × 20 cms. New York, Scripta Mathematica, 1947. Price $3.75.

Cassius Jackson Keyser has had a distinguished career as a mathematician and a leader in the world of thought. In his honor and to make his work available to the public, a committee of the Friends of Cassius Jackson Keyser has arranged for the publication of his collected works, of which this is the first volume. The essays presented are penetrating in their analysis, stimulating in their effect and a source of clear thinking in these troubled times.

The majority of the essays bear on the philosophy of mathematics, the first essay being a detailed discussion of the meaning of mathematics. The title essay concerns itself with the thesis that "the type of Mathematics found in any major culture is a clue, or key, to the distinctive character of the Culture taken as a whole." The development of this thesis forms one of the more interesting essays in the book.

Three of the essays deal with the works of individuals—Charles Sanders Peirce, William Benjamin Smith and Vilfredo Pareto, mathematicians all. Keyser offers thoughtful comments on their various writings.

This first volume of the Collected Works is an attractively prepared book which will lead the reader to peruse with pleasure the learned but lucid contents.

G. E. PETTENGILL.

MANUAL OF FOUNDRY AND PATTERN SHOP PRACTICE, by Otis J. Benedict, Jr. 361 pages, drawings and illustrations, 12 × 19 cms. New York and London, McGraw-Hill Book Company, Inc., 1947. Price $3.25.

In more and more engineering schools, the shops are being given a full measure of importance in the curriculum. The shops comprise forge, pattern shop, foundry and machine shop.

Because of the close alliance between pattern design and molding practice, Mr. Benedict's book supplies a text in the fundamental principles of these two.

The first part of the book deals with foundry practice. Fundamental principles as applied to the production of gray-iron castings are dealt with, while no attempt has been made to cover the entire ferrous and non-ferrous fields. It includes such topics as molding equipment, molding operations, cores and coremaking, the cupola, the pouring and cleaning of molds, technological control, etc.

The remainder of the book covers pattern making, and is planned so as to give the student a clearer understanding of the problems involved in the construction and use of patterns, core boxes, match plates, sweep work and follow boards. A brief description of the various saws, the characteristics of woods, the seasoning of lumber, the properties of glues, plywoods and veneers is also given.

Twenty-seven pages of problems for pattern design and pattern construction, as well as a list of visual aids, round out this comprehensive volume.

HENERY N. MICHAEL.

ELEMENTARY FLUID MECHANICS, by John K. Vennard. Second edition. 339 pages, drawings, 14 × 22 cms. New York, John Wiley & Sons, Inc., London: Chapman & Hall, Limited, 1947. Price $4.00.

This textbook is intended primarily for the beginner in the study of fluid phenomena who knows the principles of mathematics and mechanics.

Presentation of the subject begins with the discussion of fundamentals, physical properties, and fluid statics. Frictionless flow, principles of similarity and dimensional analysis are taken up next. With this background, frictional processes and pipe and open-channel flow are treated. The presentation of the principles and apparatus for fluid measurements and an elementary treatment of flow about immersed objects end the text.

In the second edition of the book, the original chapter on "Ideal Fluid Flow" has been expanded into three separate chapters dealing respectively with incompressible fluid, compressible fluid, and the impulse momentum ban..

Other changes include modernization of the chapter on pipe flow, practical pipe-line problems, discussions of the effects of velocity distribution on velocity bend and momentum, and methods for the calculation of critical depth in non-rectangular channels.

Condensation of mathematical developments and deletion of certain topics that were beyond the capacity of the beginner, or were treated elsewhere, or were obsolete, further characterize the second edition.

Problems now available for student solution number over 700.

HENRY N. MICHAEL.

ELEMENTARY NUCLEAR THEORY, by H. A. Bethe. 147 pages, drawings, 14 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.50.

In his preface Dr. Bethe, who during the war was a staff member of the Manhattan Project, states that his latest book is not meant to be a textbook of the theory of atomic nuclei. Rather, he emphasizes the central problem of nuclear physics—nuclear forces, and treats the subject empirically. Theoretical considerations of nuclear forces (the meson theory) are treated with brevity.

The theory of beta disintegration was chosen by Dr. Bethe as a second, developed and fundamental field of nuclear physics. Again, he treats the theory of the compound nucleus very briefly, judging it to be "more profitable to treat part of the theory thoroughly than to treat all of it superficially." The theories of the fission process, those of alpha radioactivity, and of nuclear systems containing from 3 to 60 nuclear particles, were left out for reasons of familiarity to the students, or of being a highly special phenomenon as in the case of fission. A table of nuclear species based on Seaborg's "Table of Isotopes" and Segre's "Isotope Chart" forms the appendix.

HENRY N. MICHAEL.

THE SELECTED WRITINGS OF BENJAMIN RUSH, edited by Dagobert D. Runes. 433 pages, 14 × 21 cms. New York, Philosophical Library, 1947. Price $5.00.

Among the leading figures in Philadelphia during the Revolution and the formative years of the United States of America, was Benjamin Rush. An outstanding physician, he was also an ardent defender of liberty and an exponent of the mitigation of social evils which bear on the poor and oppressed. His personal courage was amply demonstrated during the yellow fever epidemic of 1795. Mr. Runes has made a selection from Rush's writings which are revealing of the man and his philosophy. They are grouped in four sections: good government, education, natural and medical sciences and miscellaneous.

GEORGE E. PETTENGILL.

ELECTRIC MOTOR MAINTENANCE, by W. W. McCullough. 126 pages, illustrations, 15 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.00.

A useful manual intended to help inspectors, mechanics, electricians, and engineers take better care of the motors for which they are responsible. The material is arranged in such a manner so as to give the reader significant information on each phase of maintenance.

The section on mechanical maintenance deals with assembly, lubrication, care of sleeve, ball and roller bearings and Kingsbury thrust bearings, as well as current-collecting devices and adjustment of air gap.

The electrical maintenance section gathers the various problems of insulation: cleaning, drying, testing, as well as the materials used for insulation.

The last section includes the operation and application of the three prevalent types of electrical motors; the induction, direct current, and synchronous motors and their combinations with reduction gears and generators.

This up-to-date manual, written in simple, direct language should prove of value to the intelligent maintenance men and the interested student.

HENRY N. MICHAEL.

THE WATER SOLUBLE GUMS, by C. L. Mantell. 279 pages, drawings, 16 × 24 cms. New York, Reinhold Publishing Corp., 1947. Price $6.00.

This book is a discussion of that interesting group of hydrophilic substances usually referred to as "gums." Hydrophobic resinous materials are not discussed in this volume.

The subject matter covers the tree exudations such as arabic, ghatti, harayá, and tragacanth; the seaweed colloids such as agar, Irish moss, carrageenin, and the algiuates; the water dispersible proteins such as gelatin, hide, bone, fish, and protein, and the various albumins; and the modified celluloses.

Uses of gums in cosmetics, pharmacy, paints, textile operations, and miscellaneous applications are covered, together with much pertinent information on sources, constitution, and physical and chemical properties. A chapter is included on specifications, identification, and test.

The author has carried out a difficult task in a creditable manner in a field where the art is ahead of the science. The book is worthwhile and fascinating.

G. S. GARDNER.

AN INTRODUCTION TO CRYSTALLOGRAPHY, by F. C. Phillips. 300 pages, 500 diagrams, 14 × 22 cms. New York, Longmans, Green & Co., Inc., 1946. Price $6.50.

Mr. Phillips' introduction text will serve students whose future interest lies in the field of minerology or that of physics and chemistry as related to the crystalline kingdom in general.

The first part of the book deals with the eternal symmetry of crystals and includes such topics as: methods of projection, the seven systems, goniometry, mathematical relations et al.

The second part, on the symmetry of the internal arrangement, covers the interal symmetry, space groups, and crystal habit.

500 line drawings illustrate the text.

HENRY N. MICHAEL.

MECHANICS, by John C. Slater and Nathaniel H. Frank. 297 pages, drawings, 15 × 24 cms. New York, McGraw-Hill Book Co., 1947. Price $4.00.

ELECTROMAGNETISM, by John C. Slater and Nathaniel H. Frank. . 240 pages, drawings, 15 × 24 cms. New York, McGraw-Hill Book Co., 1947. Price $3.50.

In 1933, the authors published their well-known "Introduction to Theoretical Physics." By separating the material into mechanics, electromagnetism, and quantum theory, and publishing each separately, the authors believe that it is possible to give a somewhat better rounded treatment of each of the fields without sacrificing the essential unity of physics.

In the text on mechanics the material is presented in the order of mathematical difficulty and in that of historical development. Thus, particle dynamics, problems of oscillations and simple vector analysis are introduced first. Vibrations, wave motion, elasticity, and hydrodynamics follow. Appendices give the mathematical background to these topics.

Though at times descriptive material and descriptions of experimental results are included, the book is very definitely one of theoretical physics.

The volume on electromagnetism is developed from first principles, and also contains in its appendices enough mathematical material so that students familiar with calculus and differential equations will not have any difficulties in following the work.

The two principal directions into which electromagnetism has developed, i.e. the electromagnetic theory of light (short wave lengths) and the longer wave lengths are dealt with.

Since 1933, the meter-kilogram-second system of units has come into use and the authors use it in this book in preference to the Gaussian system. However, the appendix discusses the various systems of units, so that the reader will not be a disadvantage in consulting books using the Gaussian system.

HENRY N. MICHAEL.

PUBLICATIONS RECEIVED.

Waterbury's Handbook of Engineering, revised by H. W. Reddick, W. M. Lansford, C. O. Mackey, H. H. Higbie, H. S. Bull. Fourth Edition. 286 pages, tables and drawings, 8 × 14 cms. New York, John Wiley & Sons, Inc. 1947. Price $2.50.

Fundamentals of Engineering Mechanics, by Alvin Sloane. 379 pages, drawings, 15 × 23 cms. New York, Prentice-Hall, Inc., 1947.

Electronic Transformers and Circuits, by Reuben Lee. 282 pages, drawings and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.50.

Patent Notes for Engineers, published by RCA REVIEW Department of RCA Laboratories Division, Radio Corporation of America. 165 pages, drawings, 15 × 23 cms. Princeton, Radio Corporation of America, 1947. Price $2.50.

The Escalator Method in Engineering Vibration Problems by Joseph Morris. 270 pages, drawings and tables, 14 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.50.

Magnetic Control of Industrial Motors, by Gerhart W. Heumann. 589 pages, drawings and illustrations, 15 × 23 cms. New York, John Wiley & Sons, Inc., 1947. Price $7.50.

Elementary Engineering Thermodynamics, by Vincent W. Young and Gilbert A. Young. Third Edition. 268 pages, drawings, 16 × 23 cms. New York, McGraw-Hill Book Company, Inc., 1947. Price $3.00.

Adult Education for Homemaking, by L. Belle Pollard. Second Edition. 194 pages, 14 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.75.

Neutron Effects on Animals, by The Staff of the Biochemical Research Foundation. 198 pages, drawings and tables, 15 × 23 cms. Baltimore, The Williams & Wilkins Company, 1947. Price $3.00.

CURRENT TOPICS.

Measurement: Tool of Science and Industry.—Hugh L. Dryden, Associate Director, National Bureau of Standards, stated recently before the Instrument Society of America, "We are often asked at the National Bureau of Standards when our work on standards of measurements and problems arising in connection with their application in science and industry will be completed. Can we not establish the standards of length, mass, time, etc., once for all? It is obvious that your work and our work will never be completed as long as science and technology continue to grow, and there is no reason to expect their growth to cease. On the contrary we are on the threshold of a new age, the age of nuclear energy, and we are in the midst of a period of intensive application of scientific knowledge to industrial processes. Your activities and ours need to be expanded, not contracted, as a measure of prudent foresight. Some of the trends of development of the field of measurement are the growth in the number of things to be measured and their complexity, the need in industrial and scientific work for more accurate measurements, the remarkable increase in the application of electrical and electronic methods to all kinds of measurements, and the beneficial results from interchange and cross-fertilization among various fields of application of instrumentation.

"Some few years ago I happened to notice on the floor of our shipping and receiving room at the Bureau an incoming shipment labeled 'one quart human breath.' This was so unusual a commodity that I inquired about it. It was indeed a sample submitted for test and correctly identified, the breath of a worker in one of the plants of the Manhattan Project to be examined for content of radio-active material. On the same day there were samples of synthetic rubber, of new plastic materials, adhesives, new anti-knock fuels and other materials not in existence a few years before.

"There has been a tremendous growth in the number of things on which measurements are to be made, in the number of properties to be measured, and in the complexity of the required measurements. Applications of atomic energy will require knowledge of the neutron absorption of materials, a new property to be measured. Since this property may be controlling, materials showing desirable characteristics in this respect will have to have other properties measured such as their structural strength and rigidity at normal and high temperatures. The number of magintudes that will need to be measured is ever increasing, for it is unlimited.

"The measurements required are becoming more and more complex in character. Complex equipment has been developed to save man physical and mental labor, which can be operated properly only when its performance and condition are known. Measuring devices must be provided to translate the physical conditions to indications or records which are readily interpreted by the operator. Complexity has led to the development of remote indication so that the measurements of the performance of a whole power plant or even a whole electrical power network can be transferred to a single board where one individual can supervise the overall performance. It is but a step to introduce automatic control of the operation of the plant, to use the pseudo-sensory reactions of the measuring instruments to command control mechanisms until the desired result is obtained to the satisfaction of the measuring instrument. The whole art of servo-mechanisms is but in its infancy at the present time. The successful development of such apparatus places new and more stringent demands on the measuring elements.

"The trend in our common field of activity is thus to develop robots of great physical power, fully equipped with a highly precise and interlocked nervous system capable of all the activities of the reflex nervous system of man, often of higher precision, and in addition capable of primitive forms of judgment. You are the creators or expert users of these complex systems.

"A second noticeable trend is the demand for increased accuracy of measurement in ordinary industrial applications of instruments. Those of us at the National Bureau of Standards are perhaps more keenly sensitive to this trend because it is our duty to try to keep one order of magnitude higher accuracy than that demanded in industry. We have seen the decimal point in industrial length measurements move to the left until accuracies of one ten-thousandth of an inch and one hundred-thousandth of an inch are fairly common. Any one may purchase at moderate cost a timing device permitting the measurement of time intervals to an accuracy of one-millionth of a second. In numerous other fields it is difficult to keep ahead of the demands of scientific and industrial users of the national standards of measurements.

"Through the whole gamut of instruments of all degrees of precision from the more qualitative to the most precise, you who are users are demanding more accuracy under more and more adverse conditions, because you find the higher accuracy economically advantageous. Precise knowledge means in general a more satisfactory product, and less waste of material and manpower.

"A third trend is the increasing application of electrical and electronic methods to problems of measurement. The use of these methods permits the meeting of more rigorous requirements as regards sensitivity, accuracy, and remote indication. Electrical methods are extremely flexible and adapted to the transfer and amplification of power. Other types of energy supplied by the detector of a physical magnitude are readily transformed to electric energy. As an example of this trend, it is estimated that the number of aircraft instruments using electrical transmission and indication of non-electrical magnitudes increased from less than 5 per cent. to more than 50 per cent. between 1935 and 1945."

R. H. O.

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